VIC-EDU

Digital Image Correlation Labs

1-4

(Student Version)

Version 1.1

Laboratory 1

Basics: Using VIC-EDU System for Non-contacting Surface Measurements

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I. OVERVIEW

The purpose of this laboratory is to provide the necessary background for using the VIC-EDU system to obtain reliable measurements of surface deformation on specimens. This laboratory will cover the following aspects of using the VIC-EDU system:

- Showing and explaining the components in the VIC-EDU system
- Describing how to physically set up the VIC-EDU system for measurements
- Describing the image acquisition process using the software acquisition component VIC-Snap (EDU) to capture test images
- Demonstrating how to capture calibration images with the VIC-EDU system using the supplied calibration target and VIC-Snap (EDU)
- Showing procedure for using calibration images to complete calibration of the VIC-EDU system
- Preparing the specimen and then applying a high contrast speckle pattern with the appropriate size and spatial variation for accurate stereo image measurements
- Explaining how to set up VIC-3D (EDU) to acquire and store images of the specimen during mechanical loading
- Demonstrating how to analyze images with VIC-3D (EDU) to obtain various types of data including surface displacements and strains
- Demonstrating how to use VIC-3D (EDU) to view the measurement data
- Demonstrating how to use VIC-3D (EDU) to plot measurement data in selected formats.

After learning these aspects of the VIC-EDU system, you will use the system to obtain measurements of a specimen that has been rigidly rotated and displaced. Using these images, you will output the measured displacements and strains and discuss your results.

NOTE

Throughout these labs, we refer to the VIC-EDU system in two ways. The hardware (case, cameras, cords, calibration targets) are referred to as "VIC-EDU" and the software (the special version of VIC-3D and VIC-Snap that are designed for this system) as "VIC-3D (EDU)" and "VIC-Snap (EDU)".

II. UNBOXING THE VIC-EDU SYSTEM

The entire VIC-EDU measurement system is shipped to you in a vibration-protected case. Figure 1 shows the interior of the case as it should look when it arrives. It should include:

Figure 1: Top view of contents in VIC-EDU vibration-protected case

1. The stereo vision measurement head containing two cameras for stereo imaging, a light to illuminate specimen, and connections required to power the cameras and output data
2. Benro tripod. The stereo-vision measurement system labeled 1 is mounted on the tripod head
3. Power supply and the two required cables for connection of the VIC-EDU system to power and to user-supplied computer
4. USB Drive for installing VIC-3D (EDU) and VIC-Snap (EDU) software
5. Bottle of black ink used with supplied roller to apply the black speckle pattern
6. Roller with 0.026" dot size in a nominally random pattern
7. Ink pad
8. Calibration target. For the fixed focal distance stereo-vision measurement head, a dot spacing of 14mm is ideal for calibration of the system.

A typical VIC-EDU system (Item 1 in Figure 1) is shown mounted on a Benro tripod (Item 2 in Figure 1) in Figure 2. The included power supply (Item 3 in Figure 1) provides the necessary power for the VIC-EDU stereo cameras. Each camera has imaging resolution of 1920 vertical pixels and 1200 horizontal pixels. Exposure times can be varied between 19μs to 1s. The cameras have fixed focal length so that the system has a focused field of view of 200mm vertically and 150mm horizontally.

Manufacturer studies with the VIC-EDU system have shown the following variability in the measurements;

- in-plane displacement is +/-2μm
- out-of-plane displacement is +/- 4μm
- in-plane strains are +/- 50με

These values are essentially bounds on the measurements so that values obtained that are well above these limits will be accurate.

The VIC-Snap (EDU) image acquisition software (Item 4 in Figure 1) for the VIC-EDU system allows the user to acquire live images. When acquiring images during an experiment, the software will allow the user to acquire an image as fast as every 2s. For each experiment, the user can capture up to 100 tagged images specifically for analysis within the VIC-3D (EDU) software.

To ensure that you will have a pattern with the appropriate speckle size, the VIC-EDU system contains a bottle of ink, a rubber roller, and an ink pad (Items 5, 6 & 7 in Figure 1). The ink pad can be refilled using the supplied bottle of ink. The roller and ink pad are used to apply a speckle pattern with an average size of 0.125 inches on the specimen surface for accurate pattern matching and thus accurate displacement measurement.

Figure 2: VIC-EDU system mounted on a Benro tripod

Prior to performing an experiment, the VIC-EDU measurement system is calibrated using the VIC-3D (EDU) software and the included calibration pattern with an appropriate dot size and dot spacing (Item 8 in Figure 1). Based on manufacturer studies, the calibration target has dots with 14mm spacing.

Also included in the box and as a PDF online (link in Reference 1), but not shown in Figure 1, is the **VIC-EDU User's Manual**. The manual will be referred to throughout this laboratory, and the user is encouraged to read the entire manual prior to and when setting up the VIC-EDU system.

NOTE

Since existing laboratories already have a variety of computers available, the VIC-EDU system does not include a laptop for interfacing with the VIC-EDU measurement head. Thus, when setting up the VIC-EDU system, you should identify the laboratory computer preferred for use with the VIC-EDU system and have it available.

III. SETTING UP THE VIC-EDU MEASUREMENT SYSTEM HARDWARE

After opening the VIC-EDU measurement system case, the VIC-EDU measurement head (Item 1 in Figure 1) should be carefully removed. As shown in Figure 3, the back of the VIC-EDU measurement head has two ports. The port shown on the right side in Figure 3 is for a USB 3.0 Type A to Type B cable. The Type B end is inserted into the right side plug and the Type A end is inserted into your computer's USB 3.0 port. The left-most port is for the power cable. The VIC-EDU system uses a single power supply (Item 3 in Figure 1) to power the light and system fan.

Since all optical components, including lights, cameras, lenses, and optical filters are pre-installed in the VIC-EDU measurement head, once the measurement head is attached to the tripod and the two cables are routed to the electrical socket and computer from the measurement head, the system is ready for imaging and data acquisition. Please refer to the VIC-EDU User's Manual for details regarding attachment of the measurement head to the Benro tripod.

To obtain focused images of a specimen, the manufacturer's studies for the fixed focal length VIC-EDU system have shown that the distance from the front of the VIC-EDU head to the specimen surface is approximately 0.50m.

Figure 3: Photograph of back panel for VIC-EDU measurement head. Rectangular USB cable is for the cameras. Round cable is the power supply connection

CAUTION

Though the VIC-EDU measurement head is sturdy, if the unit is inadvertently dropped or impacted, then it is possible for the internal optical components to be shifted or damaged, requiring the unit to be returned to CSI for replacement or repair. PLEASE handle the unit with care and return the unit to the case whenever it is not in use.

IV. INITIATING THE SOFTWARE FOR IMAGE ACQUISITION

Once the VIC-EDU measurement head has been powered up and connected to the laboratory computer, the VIC-EDU measurement head should be mounted to the Benro tripod and arranged in the laboratory to begin acquiring images. The images do not have to be of the experimental specimen at this point.

Once positioned to view an object that is in focus, you are ready to acquire and store images. To do this, install the VIC-3D (EDU) software on your computer. After installing the software, initiate image acquisition by opening VIC-Snap (EDU) installed on the computer. Since the VIC-EDU system has pre-specified camera types (Point Grey for most systems), you should see two live image feeds from each camera in the system.

There are several imaging considerations to investigate as soon as the live images are visible on your laptop, and these are highlighted in the following sub-sections.

IV.a. Centering specimen in both cameras

To acquire measurement data over the largest region on the specimen, it is important that the same specimen region is viewed by both cameras. One way to do this is to use internal functions within VIC-Snap (EDU) on pages 3 and 4 of the User's Manual, and is described briefly here.

- Identify a specific point and locate its pixel position in both images with the VIC-3D (EDU) cross-hair.
- Move specimen using the ROTATION function in VIC-Snap (EDU) and by manual translation of the specimen, system, or both until the point is approximately centered (pixel (600, 960)) in both images.
- Ensure the region remains in focus in both cameras by visually assessing the sharpness of the images. Use the focus tool in VIC-3D (EDU) to achieve the sharpest focus possible.

IV.b. Optimizing lighting without overexposure or underexposure

To obtain accurate measurements using the VIC-EDU system, the image of a surface of the specimen must not be over-exposed (too bright) or under-exposed (too dark). Appropriate exposure is obtained by controlling both the exposure time and the orientation of the polarizers on the lenses. This is discussed in detail on pages 3-5 of the User's Manual and you should employ these procedures while viewing the live images, varying both exposure time and polarizer orientation to see how the specimen image is modified.

NOTE

In addition to maximizing the region over which measurements can be obtained, centering of a common region in both cameras also provides another benefit; reducing the potential for optical distortions to affect the measurements. Additional discussion of this issue is available in a variety of publications, including [2].

V. VIC-EDU SYSTEM CALIBRATION

Now that the VIC-EDU system is operational, it is possible to acquire and store images in system memory. To do so, you will use the supplied calibration target (Item 8 in Figure 1) with the VIC-EDU system and the VIC-3D (EDU) software. Details regarding the calibration process are shown in the VIC-EDU User's Manual on pages 5-8. Briefly, the operational VIC-EDU system is placed in position and the calibration target is placed about 0.50m in front of the VIC-EDU system to view a live image.

After centering, focusing, and optimizing exposure, images such as shown in Fig. 4 are obtained for the calibration target. Once high quality (e.g., high contrast, sharply focused) images are obtained, the calibration process for the VIC-EDU system is initiated. Key points in the calibration process are as follows;

- Rotate target about three perpendicular axes
  - Rotate without defocusing the grid pattern
- Always be sure that the three special dots are present in BOTH images
- Acquire between 30-50 images of the target in different, focused orientations.

After acquiring all calibration images, the calibration process can be completed and the calibration parameters obtained following the process described in the VIC-EDU User's Manual. It is important to emphasize that the "Score" value must be within the recommended range to ensure reliable measurements with the system. If the "Score" is too high, refer to the User's Manual, pgs. 7 and 8, to correct imaging issues prior to repeating the calibration process.

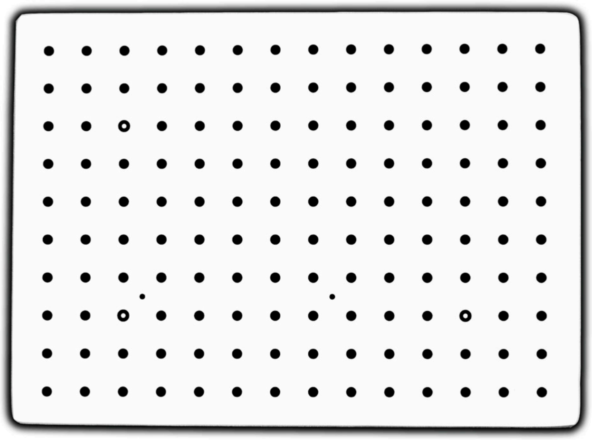

Figure 4: Photograph of a typical calibration target used to calibrate stereo-vision measurement systems such as VIC-EDU

VI. EXPERIMENTAL PREPARATIONS

After successfully calibrating the VIC-EDU system, it is time to prepare the test specimen. The process is described in detail in the Correlated Solutions Inc. application note AN-1701 [3]. The process is briefly summarized here:

- Use coarse and fine grit sandpaper to smooth surface and remove unwanted adherent materials
- Use CSM-2 degreaser or similar cleaning agent to remove any residual oils and metal particles left on the surface by sanding. Repeat this as needed.
- Allow surface to dry and then paint the specimen surface. DO NOT paint specimen in same room where imaging is performed, as it will degrade the optical imaging elements. Typically, surface is initially painted white.
- Use only enough paint to coat surface and minimize reflections; DO NOT OVERPAINT SURFACE.
- After the paint has dried, apply a random dot pattern with size of 0.026″ (0.635 mm) using the supplied roller and ink pad. It should require approximately 2-3 passes in order to achieve the 50/50 ratio of white to black that we are aiming for. Remember, do not smear the pattern as this can degrade quality of measurements.
- Use care to apply distinct dots. To improve contrast, you may want to perform multiple passes with the roller. Do not try to align the passes, just apply pattern on top of the previous one, checking the pattern using VIC-3D (EDU) tools.

Please refer to [3] for additional details regarding effective specimen surface preparation.

VII. PERFORMING EXPERIMENT WITH IMAGE ACQUISITION

Once calibration is complete, it is time to acquire images of the as-patterned specimen and store them. To do so, click on *Edit Files* in VIC-3D (EDU) and rename the image prefix to indicate test images are being stored. To begin storing images with this name, you can either (a) do it manually using the Spacebar or the Capture icon in VIC-3D (EDU) or (b) use the *Time Capture* option and set the time between image acquisitions. Refer to the User Manual, page 6, for additional details.

NOTE

It is true that, in principle, a calibrated VIC-EDU measurement system can be moved to a new position without affecting the calibration. Since excessive handling of the system always has the possibility of affecting the internal components, it is recommended that the system be placed in its final position for the experiment and the calibration target placed just in front of the location where the specimen will be located. In this way, calibration can be performed without moving the VIC-EDU system. Once the calibration is complete, the specimen is simply put in place and the experiment performed.

VIII. IMAGE ANALYSIS FOR DEFORMATION MEASUREMENTS

After acquiring experimental images, the VIC-EDU software is used to extract the deformation measurements by performing image analysis using the calibration parameters. The software has incredible flexibility in how to perform the analysis. Many of the capabilities and options are described on pages 9-19 of the VIC-EDU User's Manual. After importing your images using the Speckle Images icon or via the Project file menu, key parameters needed for all analyses are as follows:

- Region of Interest (ROI): The region on the specimen where data is to be obtained is to be selected. See pgs 8-10 in User's Manual.
- Subset size selection: All subsets are square, so a single pixel dimension is requested when making the selection. See pgs 8-10 in User's Manual.

- Step size selection: The pixel distance in each direction between subsets that are analyzed within the ROI. See pgs 8-10 in User's Manual.
- Initial Guess: In most cases, the software does not need an initial guess. However, if the motions are large due to the loading, then the VIC-EDU software may require an initial guess before beginning the analysis. The process for doing this is described on pgs 9-11 in the User's Manual.

With this information, the VIC-3D (EDU) image analysis software can be initiated using the ***Start Analysis*** icon. This will open a window that gives additional analysis options, as discussed on pgs 10-13 in the User's Manual. Please refer to the discussion provided in the User's Manual to make your selection. Once all analysis options have been selected, the analysis is initiated via the Run button. After analysis has been completed, the measurement data is now available for post-processing.

Post-Processing and Plots
A rather complete discussion of various output variables and how to display the results is given on pages 14-19 in the User's Manual. You should review this in detail prior to initiating detailed processing of the data.

IX. LABORATORY RESULTS TO BE PRESENTED

1. Speckle pattern a planar specimen using the procedure described above. Once patterned, use the VIC-EDU measurement system to acquire focused images of the specimen.
   - Take a photograph showing both the VIC-EDU measurement system and the specimen and show in your report.
   - Take both images of the specimen that are recorded by VIC-EDU and show these in the report.
2. An important parameter in optical imaging is the distance you can move a specimen towards and away from the cameras while continuing to acquire well-focused images. Known as Depth Of Field (DOF), you will experimentally estimate DOF for a planar specimen. See Fig. 5 for description.

- Place specimen at about 0.50m from measurement head
- Orient specimen to be roughly parallel to centerline of measurement head
- Place ruler perpendicular to specimen surface
- Use live imaging
- Move specimen TOWARDS measurement head until you can see slight blurring. Record ruler reading, d_f
- Move specimen AWAY from measurement head until it becomes focused and then slightly blurred. Record ruler reading, d_b
- $DOF = +/- \frac{|d_f - d_b|}{2}$
- Repeat this process 10 times. Record data in table and include in report.

Figure 5: Schematic of experimental setup

3. Suppose a specimen of horizontal length, L, is rotated by an angle, θ, about an axis near the center of the front surface, as shown on the right. Determine the maximum angle that the specimen can be rotated so that all points on the front surface are within DOF determined in Question 2.

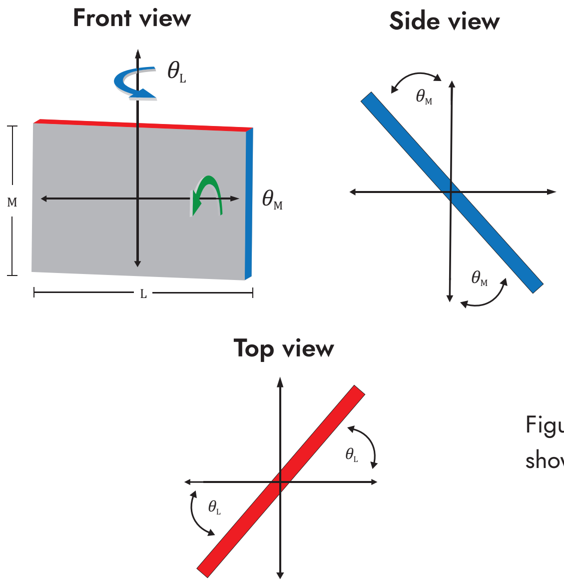

Figure 6: Schematic of specimen showing rotations θ_L and θ_M for step 3

- Determine the maximum angle that the target can rotate for both θ_L and θ_M, where L and M are the actual dimensions of the calibration target provided with the VIC–EDU system. See Fig. 6 to provide additional guidance. You should measure both L and M if these dimensions are not engraved on the target. You may want to consider equations such as the following.

$$\theta_L = \sin^{-1}\left(\frac{2\ \mathrm{DOF}}{L}\right) \quad \theta_M = \sin^{-1}\left(\frac{2\ \mathrm{DOF}}{M}\right)$$

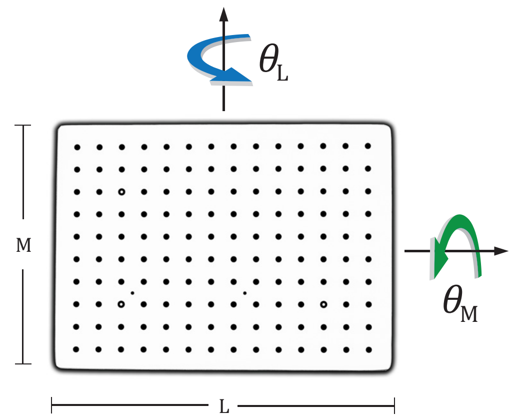

Figure 7: Schematic of calibration grid and rotations

4. Use the as-supplied calibration target and perform a complete calibration of the VIC-EDU system. A typical target is shown in Figure 7. Include the following in your report.
   - Number of calibration images
   - One of the calibration target images
   - Calibration reported by the VIC-3D (EDU) software, including two screenshots similar to the following, if not already in the report. The project files on right should contain camera parameters for both camera 1 and camera 2, as well as the transformations. Examples are shown below in Figures 8 & 9 for your use.

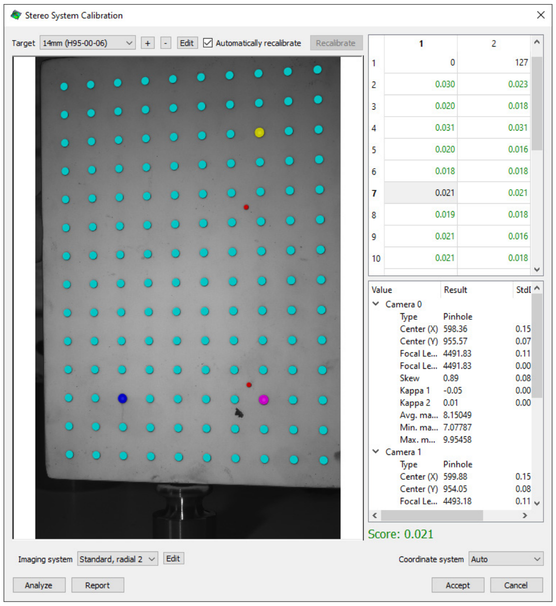

Figure 8: Calibration dialog window

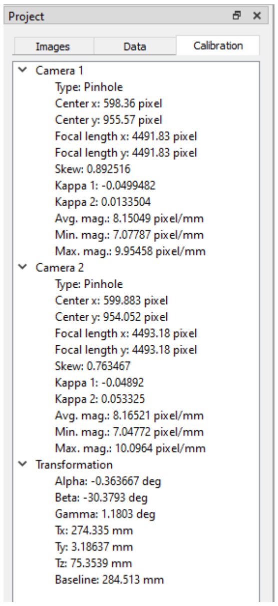

Figure 9: Calibration tab in project workspace

5. Using the VIC-EDU system, acquire two pairs of test images of the as-patterned specimen after it has been centered, focused and properly exposed. Both pairs of images should be taken without moving the specimen. Include one pair of test images in the report.

- The two image pairs are to be analyzed using the VIC-3D (EDU) software to obtain and report data over a 500 x 500 pixel region centered in the image. Fill in the table below to provide a list of all options/parameters used in the analysis
- ROI: ~500 x 500 pixels; Subset size; Step size
- Found in Start Analysis dialog: Subset Weights; Interpolation method; Correlation criterion
- Found in Start Analysis dialog: Consistency Threshold; Confidence Margin; Matchability Threshold
- Found in Start Analysis dialog: Auto Plane Fit was enabled; Confidence Margins computed or not; Strain filter size and type of strain calculated.

The data you are to report are as follows:

- 2D full-field plot of the SHAPE of the surface ($z(x, y)$) for the ROI after enabling the Auto Plane Fit option, where x and y are nominally horizontal and vertical lines, respectively, on the specimen and z is orthogonal to both x and y lines
- 2D full-field plot of the out-of-plane displacement field, $W(x, y)$, for the ROI
- Extraction plot for the Lagrangian strain e_{xx} with a filter size of 15 for a horizontal line passing through the center of the ROI
    - Recalculate strain for e_{xx} using a filter size of 5 (refer to pages 14 – 19 in the User's Manual)
    - Show extracted line with both filter results on same extraction plot
- Extraction plot for the Von Mises strain with a filter size of 15 for a vertical line passing through the center of the ROI
- Recalculate strain for e_{xx} using a filter size of 5 (refer to pages 14 – 19 in the User's Manual)
- Show extracted line with both filter results on same extraction plot.

Lab 1 - Stereo DIC Image Analysis Parameters

Region of Interest (ROI) (pixels x pixels)	
Field of View (pixels x pixels)	
Digital magnification (pixels/mm)	
Subset size (pixels)	
Subset spacing (pixels)	
Correlation criteria used	
Subset weighting	
Interpolation method	
Consistency threshold (pixels)	
Confidence margin (pixels)	
Matching threshold (pixels)	
Strain metric type	
Strain filter size (N x N pixels^2)	
Strain filter type	
Auto-plane fit	Yes or No

NOTES

X. REFERENCES

1. VIC-EDU User's Manual, Correlated Solutions Incorporated, www.correlatedsolutions.com
2. MA Sutton, JJ Orteu, and HW Schreier, Image Correlation for Shape Motion and Deformation Measurements; Theory and Applications, Springer (2009) ISBN: 978-0-387-78746-6.
3. Application Note AN-1701:Speckle Pattern Fundamentals, http://www.correlatedsolutions.com/support/index.php?/Knowledgebase/Article/View/80/1/speckle-pattern-fundamentals.

Laboratory 2

In-Plane Beam Bending with the VIC-EDU Measurement System

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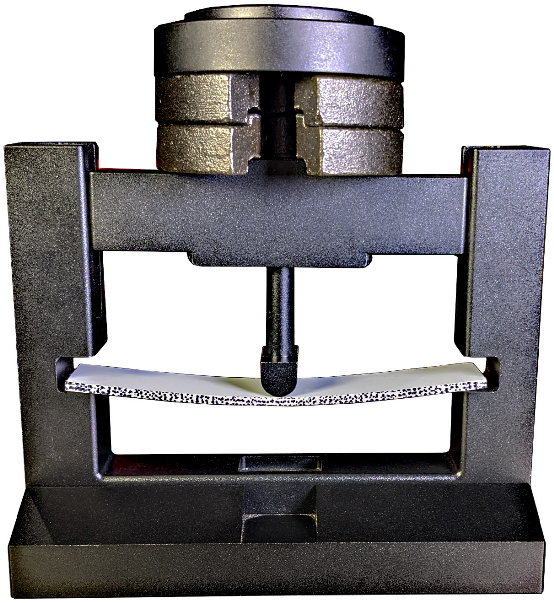

Laboratory 2

I. OVERVIEW

The purpose of this laboratory is to utilize optical imaging systems with stereo digital image correlation within VIC-EDU to make full-field, non-contacting, surface deformation measurements with the educational measurement system for a simply supported beam specimen subjected to centrally located loading. As part of this laboratory, students will be provided basic theoretical background to predict the surface displacements and strains that they will measure. To obtain the measurements, students will learn how to (a) prepare the specimen surface and apply a usable, high-contrast speckle pattern, (b) arrange the VIC-EDU system to acquire images that can be used for stereo analysis, (c) calibrate the VIC-EDU system for stereo DIC, (d) acquire and store images during mechanical loading of the specimen, (e) perform post-processing to obtain results using the stereo images and (f) compare the experimental results with the theoretical solutions.

NOTE Throughout these labs, we refer to the VIC-EDU system in two ways. The hardware (case, cameras, cords, calibration targets) are referred to as "VIC-EDU" and the software (the special version of VIC-3D and VIC-Snap that are designed for this system) as "VIC-3D (EDU)" and "VIC-Snap (EDU)".

II. BEAM THEORY

A specimen for which the well-known Euler-Bernoulli beam theory is applicable is shown schematically in Fig. 1a, where the theoretical formulation generally requires that $L >> h >> t$. Typical ratos for $\frac{h}{L}$ and $\frac{t}{h}$ are on the order of 0.5, 0.33, 0.25 or even lower, depending upon experimental limitations for the application. If these conditions are met, and the deformations are small for the centroidal coordinate system shown in Fig. 1b, we can write the following expression for the vertical displacement of the beam.

$$(1) \qquad \frac{d^2 v}{dx_c^2} EI_{z_c z_c} = M_{z_c}(x_c)$$

where E is the elastic modulus (units of Pa or psi) of the material; $I_{z_c z_c}$ is the area moment of inertia of the beam cross-section about the centroidal z_c axis; v is the vertical displacement of the centroidal $z_c - x_c$ surface at distance x_c along the beam; $M_{z_c}(x_c)$ is assumed to be a positive internal reaction moment as a function of the distance, x_c, along the beam; $F_{y_c}(x_c)$ is the vertical shear force on the cross-section; C is the centroid of the beam cross-section. The specimen provided with the VIC-EDU system has an estimated modulus of elasticity, E ~ 6 GPa. As described in a NOTE in Section VI of this lab, there may be variability in E due to manufacturing differences for the as-supplied specimen material.

For the rectangular beam shown in Fig. 1b, the location of the centroid is $(x_c, \frac{h}{2}, \frac{t}{2})$ relative to the bottom left corner of the beam (see Fig. 2). The origin for the centroidal coordinate system can be chosen anywhere along x_c for the beam specimen. However, in practice, the origin is oftentimes chosen at the left end of the beam specimen so that x_c is positive along the entire length of the beam.

In addition to the vertical displacement, $v(x_c)$ in Eq. 1, when cross-sections shown in Fig. 2 remain planar during mechanical loading, then there is a normal stress, $\sigma_{x_c x_c}$, on each cross-section that is given as;

$$(2) \qquad \sigma_{x_c x_c} = \frac{-M_{z_c}(x_c) y_c}{I_{z_c z_c}}$$

where y_c is vertical distance from the neutral surface. The stress distribution corresponding to Eq. 2 is shown in Fig. 3.

If the beam is assumed to be loaded only on the top and bottom surfaces in the vertical directions and the beam is an isotropic, homogeneous, linear elastic material, then the corresponding strains along the x_c direction can be written, Eq. 3.

$$\varepsilon_{x_c x_c} = \frac{-M_{z_c}(x_c)\,y_c}{EI_{z_c z_c}} \tag{3}$$

Thus, horizontal lines above the centroid ($y_c > 0$) on the front surface will be compressed and horizontal lines below the centroid ($y_c < 0$) will be stretched. Furthermore, according the Eq.(3), the strain $\varepsilon_{x_c x_c}$ varies linearly with vertical distance from bottom to top of the beam in the same way as the stress distribution varies in Fig. 3. In this laboratory, the external load will be applied at or very near the center of the beam, $x_c \approx \frac{L}{2}$.

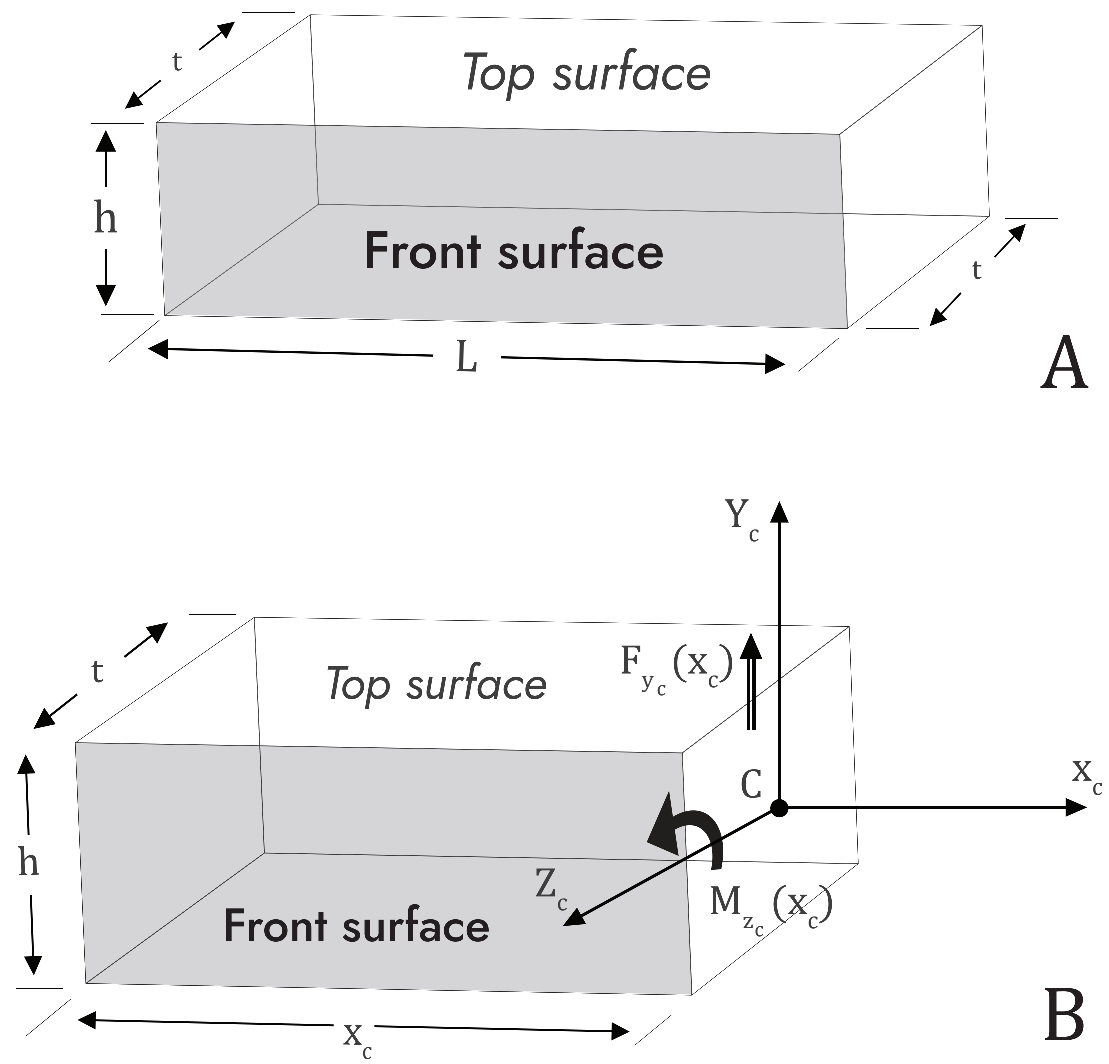

Figure 1: Beam specimen schematic with coordinate system and positive directions for internal static reactions

II.a. Implications of Theory for Measurements on Front Surface of Beam

As shown in Eq. 1, points along the centerline of the beam will move vertically, either down (negative movement) or up (positive movement) when a mechanical load is applied and a bending moment is developed within the beam. Thus, you should expect that the front surface will move vertically and you will measure how the vertical displacement, $v(x_c)$, changes as you move horizontally across the beam.

In addition to vertical movement of lines on the front surface, Eq. 3 indicates that points on the front surface will be stretched on one side of the centroid and compressed on the other side of the centroid. Thus, if there is sufficient spatial resolution in the measurements across the height of the beam, there should be a variation in the measured strain $\varepsilon_{x_c x_c}$ that is approximately linear in the y_c direction as you move up and down on the front surface of the beam.

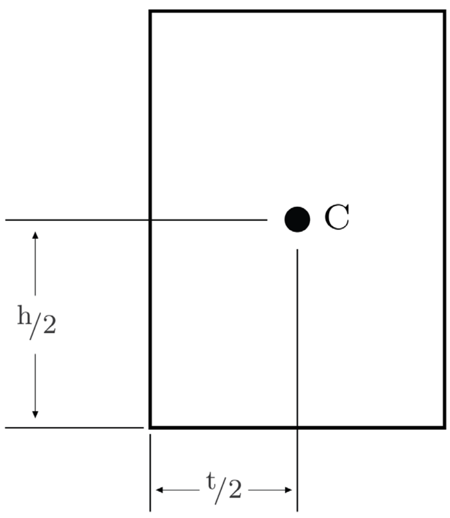

Figure 2: Rectangular cross-section and centroid location

Another important implication of the theory is that you can PREDICT or ESTIMATE both the vertical displacements and axial strains that will occur in your specimen prior to loading. If the elastic modulus of the beam, dimensions of the beam, end conditions (how beam is supported at ends) and the range of mechanical loads to be applied are known, then static equilibrium can be used to obtain the internal moment and Eq. 1 can be solved for the vertical displacement along the beam. The expression for $v(x_c)$ that is obtained can be used to determine whether the experimental displacements should be large enough to be measured by the VIC-EDU system for some of the loads in the range that is to be applied. You do this by comparing your predictions to the as-supplied lower bound (+/- 0.02mm for in-plane displacement). This is extremely useful in practice as you will know a priori whether you are well above the lower bounds and the experiment will be productive or not.

III. EXPERIMENTAL PREPARATIONS

III.a. Specimen Preparation

To make accurate DIC measurements using images of a specimen, the specimen must be prepared and the FRONT SURFACE patterned with high contrast speckles having the appropriate size and an approximately random distribution across the field of view. The procedures to do this are detailed in Lab 1 and in Application Note AN-1701 [3] and are briefly summarized here:

- Use coarse and fine grit sandpaper to smooth surface and remove unwanted adherent materials
- Use CSM-2 degreaser or similar cleaning agent to remove any residual oils and metal particles left on the surface by sanding. Repeat this as needed
- Allow surface to dry and then paint the specimen surface. DO NOT paint specimen in same room where imaging is performed, as it will degrade the optical imaging elements. Typically, the surface is initially painted white
- Use only enough paint to coat surface and minimize reflections; DO NOT OVERPAINT SURFACE
- After the white paint has dried, apply a random black dot pattern with size of 0.026″ (0.635 mm) using the supplied roller and ink pad. Patterning should require approximately 2-3 passes in order to achieve the high contrast 50/50 ratio of white to black that is best. Do not try to align the roller passes, as attempts to move the roller around during patterning will smear or blur the dots, potentially degrading the quality of stereo DIC measurements. When done, image the front surface and check the pattern with VIC-3D (EDU) tools.

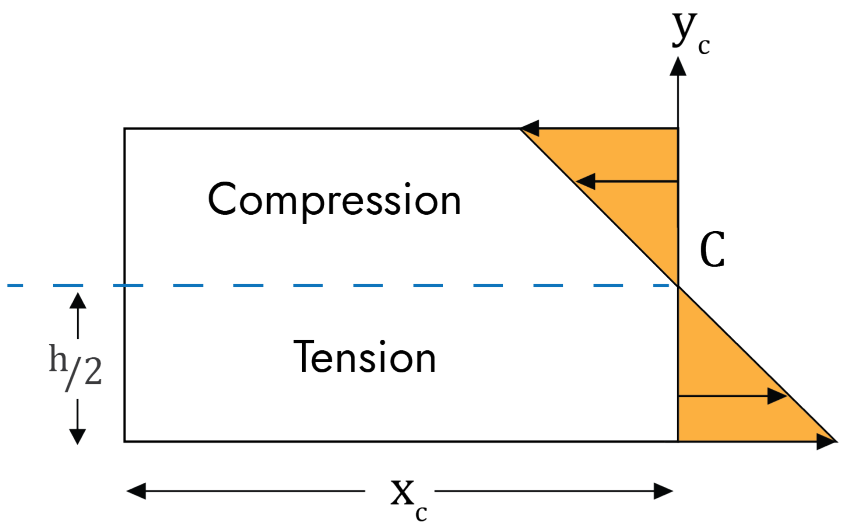

Figure 3: Schematic of axial stress distribution on vertical cross-section of beam

III.b. Installing Specimen in Loading System

Once the specimen has been prepared, the specimen will be placed in a mechanical loading frame so that the boundary supports for the beam are known with reasonable approximation. For example, if the beam is placed on two "knife edges" or rollers, then these points could be considered to be SIMPLE SUPPORTS for static analysis of the beam. Vertical loading in y_c direction will be applied on the top surface, preferably by a loading fixture that applies load along a Z_c-oriented line that spans the thickness of the beam.

III.c. VIC-EDU System Preparation

Once the specimen is in the loading frame, the tripod should be located in front of the specimen and the VIC-EDU system firmly attached to the tripod mounting head. At this point, the VIC-EDU system should be powered up and oriented to obtain images of the front surface of the specimen. Once the system is operational, the procedures outlined in Lab 1 and the User's Manual for focusing the specimen and preparing to acquire images should be used to locate the system in a position where focused images can be obtained. At this point, calibration of the system should be performed.

III.d. VIC-EDU System Calibration

Once the VIC-EDU system is mounted to the tripod and placed in position where the specimen is in sharp focus, system calibration is performed. Though in principle the entire system could be picked up and moved carefully to a new location for calibration, and then moved back into position for viewing the specimen without affecting the measurements, such movements may introduce changes in the optical imaging system that are not readily apparent and can affect the accuracy of the results. To minimize the potential for such errors, it is recommended that you calibrate in front of the test specimen. There should be enough depth of field to allow for this. Lighting provided within the VIC-EDU measurement head should provide bright, clear images of the as-supplied calibration target.

When performing calibration, it is recommended that the target occupy 80-90% of the field of view. For this experiment, the supplied target dot pattern has black dots with a 14mm spacing on a white background. Furthermore, it is required that all three of the black dots containing a central white dot be within the field of view of both cameras. These dots provide the necessary information for estimating rotation of the planar target.

To perform calibration, the target should be in focus for both cameras. Since the focus is fixed on the VIC-EDU system, the target should be located approximately 0.50m in front of the system. Then, the target is rotated several times and focused images acquired. The rotations should include motions about three separate orthogonal axes to ensure accurate identification of all calibration parameters. Typically, between 25 and 50 calibration images are acquired. Please refer to Lab 1 and the User's Manual for additional information regarding calibration and processing of the calibration images.

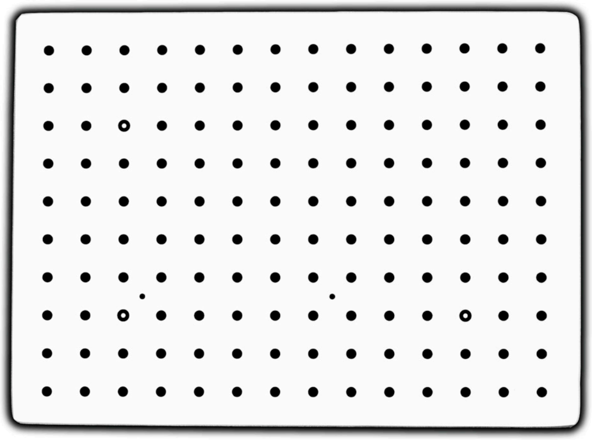

Figure 4: Photograph of a typical calibration target used to calibrate stereo-vision measurement systems such as VIC-EDU

IV. PERFORMING EXPERIMENT WITH IMAGE ACQUISITION

Once calibration is successfully completed in front of the specimen, the target is removed so the front surface of the beam specimen already placed in the loading frame is visible and in focus.

- Read Lab 1 and VIC-EDU User's Manual regarding the procedure for initiating VIC-3D (EDU) to acquire and store image pairs for this experiment.
- Acquire several image pairs of the speckled front surface of the specimen in the unloaded state.
- Apply the first increment of load/displacement and acquire two to three pairs of images, recording the image numbers and the corresponding applied load/displacement.
- Continue load/displacement application and image acquisition process until you reach the pre-identified maximum load or displacement for this specimen.
- Terminate the image acquisition process and the experiment via the process described in Lab 1, e.g., close VIC-Snap (EDU).
- Unload the specimen.

At this point, the experimental process is complete and the deformation data is embedded in the image pairs that were acquired during the experiment. To extract full-field deformation data, the VIC-3D (EDU) software is initiated to analyze the images. Please refer to the detailed description in Lab 1 for how to initiate VIC-3D (EDU) and extract displacement and strain data.

- Step size selection: The pixel distance in each direction between subsets that are analyzed within the ROI. See pgs 8-10 in User's Manual.
- Initial Guess: In most cases, the software does not need an initial guess. However, if the motions are large due to the loading, then the VIC-3D (EDU) software may require an initial guess before beginning the analysis. The process for doing this is described on pgs 9-11 in the User's Manual.

With this information, the VIC-EDU image analysis software can be initiated using the ***Start Analysis*** icon. This will open a window that gives additional analysis options, as discussed on pgs 10-13 in the User's Manual. Please refer to the discussion provided in the User's Manual to make your selection. Once all analysis options have been selected, the analysis is initiated via the Run button. After analysis has been completed, the measurement data is now available for post-processing.

Post-Processing and Plots

A rather complete discussion of various output variables and how to display the results is given on pages 14-19 in the User's Manual. You should review this in detail prior to initiating detailed processing of the data.

V. POTENTIAL SOURCES OF DIFFERENCES BETWEEN PREDICTIONS & MEASUREMENTS

For the vertical displacement, $\mathrm{v}(x_c)$, the analytical results using Eq. 1 require you to assume specific boundary conditions at $x_c = 0$ and $x_c = L$. If you assumed that the $\mathrm{v}(0) = 0$, but the support actually moved downwards during loading, then this will affect the comparison. Since this is also true at the right end of the beam, the entire $v(x_c)$ curve may have to be shifted up or down so that the ends match the assumed boundary conditions. Such motions are RIGID BODY MOTIONS (RBMs) and are quite common in experiments. The presence of RBMs may need to be accounted for when looking at the comparison of experimental and theoretical displacements. Additional issues that could affect the quality of the comparison between theory and experiment include:

- Slight shifts in the load position relative to the assumed location ($x_c = \frac{L}{2}$)
- Accuracy of the measured geometric lengths
- Accuracy of the assumed modulus of elasticity
- Slight temperature changes
- Potentially other factors that are not noted here (e.g. vibrations).

Interestingly, since the strains obtained from VIC-EDU are defined in a way that eliminates the effect of RBM, if a VIC-EDU system has sufficient spatial resolution across the height, the bending strain field measurements would be accurate and independent of RBM.

An additional source of potential errors is the way in which the experimental data is processed. For example, the experimental strains obtained from VIC-EDU use the measured displacements in a region, with a "finite-size strain filter" to estimate the strains at a point. The strain filter has various sizes over which it acquires the strain. This process may introduce slight shifts in the strain relative to the actual strain at the corresponding point on the specimen.

VI. LABORATORY RESULTS TO BE PRESENTED

1. Measure the specimen geometry (L, h, t) several times with a micrometer or similar device and provide a table of the individual results, mean value ($\bar{X}$) for each dimension and standard deviation, S_x for each dimension. The basic equations are given below;

$$\bar{X} = \sum_{i=1}^{n} \frac{x_i}{n} \quad ..Mean\ Value$$

$$S_x = \left[\sum_{i=1}^{n} \frac{(x_i - \bar{X})^2}{n-1}\right]^{\frac{1}{2}} \quad Standard\ Deviation\ (S.D.)$$

Note: The range on the modulus of elasticity, E, for the test material is 3 GPa ≤ E ≤ 9 Gpa, with the expected value being 6 Gpa.

2. Assuming beam boundary conditions that are consistent with the actual experiment just performed, solve Eq. 1 to obtain the predicted displacement of the centerline of the beam, $v(x_c)$. Show your analysis and final equation for $v(x_c)$. As a hint, your equations should give $v\left(\frac{L}{2}\right)=-\frac{PL^3}{48EI_{z_cz_c}}$ if $v(0) = v\ (L) = 0$.

2.a. Verify that your $v(x_c)$ equation satisfies your assumed boundary conditions at left side and right side of the beam. Show the equations with these values inserted to demonstrate results.

3. Using the equation for $v(x_c)$ in Question 2 with the measurements obtained in Question 1, obtain equations for the mean value and standard deviation in the predicted vertical displacement at the center of the beam, $x_c = \frac{L}{2}$. Do not consider variations in load or load position, only variations in the geometric variables measured in Question 1.

4. Using the VIC-EDU system, acquire two pairs of test images of the as-patterned specimen after it has been centered, focused and properly exposed, but without any load. Both pairs of images should be taken without moving the specimen. Include one pair of test images for the 3 point bend specimen in the report.
   - The two image pairs are to be analyzed using the VIC-EDU software to obtain and report data over a central region of the specimen. As you are setting up the analysis, fill in the table below (same format as in Lab 1) to provide a list of all options/parameters used in the analysis. Note that this table should be filled out for all stereo DIC experiments, without exception, to have a record of the analysis parameters being used.
   - Plot the initial shape of the specimen, $Z(x, y)$, to see whether it is nominally flat or not.

5. Estimate the location of the beam centerline on your image and then extract the vertical displacement data, V_i, using VIC-EDU at "i" discrete locations along the length of the beam from $0 \le x_c \le L$ for six applied loads.

$$P = 0, \frac{P_{max}}{5}, \frac{2P_{max}}{5}, \frac{3P_{max}}{5}, \frac{4P_{max}}{5}\ and\ P_{max}..$$

Here, we assume that $x_c = 0$ corresponds to the centerline of the beam above the left support and $x_c = L$ corresponds to the centerline of the beam above the right support.

5.a. Determine the specimen shape in the unloaded state.

5.b. For each of the applied loads, specimen dimensions and elastic modulus of the beam, determine whether the predicted, theoretical displacement at each location x_i is above the lower bound limit of the VIC-EDU measurement system.

6. Plot the predictions and experimental data, $v(x_i)$ and $V(x_i)$, at each x_i location where experimental data was obtained for each of the six applied loads on the same graph and discuss the results.

7. Select vertical lines that lie along the y_c direction at $x_c = 0.40L$ and $x_c = 0.60L$. For these lines, use Eq. 3 to predict the strain ε_{xx} at several y_j locations from $-\frac{h}{2} < y_c < \frac{h}{2}$ for $P = P_{max}$. Note that you must use your static analysis to obtain moment $M_{z_c}(x_c)$ at these x_c-locations.

7.a. Discuss whether the strain distribution you calculated is measurable with the VIC-EDU system or not.

8. Summarize results for displacements and for analytic strain predictions.

Lab 2 - Stereo DIC Image Analysis Parameters

Region of Interest (ROI) (pixels x pixels)	
Field of View (pixels x pixels)	
Digital magnification (pixels/mm)	
Subset size (pixels)	
Subset spacing (pixels)	
Correlation criteria used	
Subset weighting	
Interpolation method	
Consistency threshold (pixels)	
Confidence margin (pixels)	
Matching threshold (pixels)	
Strain metric type	
Strain filter size (N x N pixels^2)	
Strain filter type	
Auto-plane fit	Yes or No

NOTES

VII. REFERENCES

1. VIC-EDU User's Manual, Correlated Solutions Incorporated, www.correlatedsolutions.com
2. MA Sutton, JJ Orteu, and HW Schreier, Image Correlation for Shape Motion and Deformation Measurements; Theory and Applications, Springer (2009) ISBN: 978-0-387-78746-6.
3. Application Note AN-1701:Speckle Pattern Fundamentals, http://www.correlatedsolutions.com/support/index.php?/Knowledgebase/Article/View/80/1/speckle-pattern-fundamentals.

Laboratory 3

Out-of-Plane Beam-Column Bending with Stereo DIC Measurements using VIC-EDU

TABLE OF CONTENTS

I. OVERVIEW

The purpose of this laboratory is to utilize optical imaging systems with stereo digital image correlation software VIC-EDU within the educational measurement system to make full-field, non-contacting surface deformation measurements for a beam-column specimen with simple end supports that is subjected to a compressive load. As part of this laboratory, students will be provided with the basic theoretical background. To obtain the measurements, students will (a) prepare the specimen surface and apply a usable, high-contrast speckle pattern on the surface of the test specimen, (b) calibrate the VIC-EDU system to acquire images that can be used for stereo analysis, (c) configure the VIC-EDU measurement head to obtain image pairs during mechanical loading, (d) acquire and store images during mechanical loading of the specimen, (e) perform post-processing to obtain results using the stereo images and (f) compare the experimental results to theoretical or hybrid experimental-theoretical solutions and discuss your findings.

II. BEAM-COLUMN THEORY

A specimen for which the well-known Euler-Bernoulli beam theory is applicable is shown schematically in Fig. 1 (see pg. 2), where the theoretical formulation generally requires that L >> h >> t. Typical ratios for $\frac{h}{L}$ and $\frac{t}{h}$ are on the order of 0.5, 0.33, 0.25 or even lower, depending upon experimental limitations for the application. If these conditions are met, and the deformations are relatively small, then the bending moment is related to the curvature of the beam in a manner that is similar to Eq. 1 in Lab 2. In this laboratory experiment, there will be an axial compressive load and the bending moment will be about the Y_c as shown in Fig. 1.

During loading, the beam will deflect in the Z_c direction, as shown schematically in Fig. 1. Also shown in Fig. 1, the beam slope is positive so that the curvature is negative along the x_c direction. Since $M_{y_c}(x_c)$ is positive as shown, the corresponding moment-curvature equation is written

$$EI_{y_c y_c}\frac{d^2w}{dx_c^2} = -M_{y_c}(x_c) \tag{1}$$

where E is the elastic modulus (units of Pa or psi) of the material; $I_{y_c y_c}$ is the area moment of inertia of the beam cross-section about the centroidal Y_c axis; $w(x_c)$ is the out-of-plane displacement of the centroidal $X_C - Y_C$ surface at distance x_c along the beam; $M_{y_c}(x_c)$ is the internal reaction moment about the Y_C axis as a function of the distance, x_c, along the beam. The direction shown in Fig. 1 for M_{Yc} is assumed to be positive. In addition to the internal reaction moment, there is an axial compressive force, P in Fig. 1, on the $Y_C - Z_C$ cross-section passing through the centroid, C, of the beam-column cross-section.

The specimen provided with the VIC-EDU system has an estimated modulus of elasticity, E ~ 6 GPa. As described in a NOTE in Section VI of this Lab, there may be variability in E due to manufacturing differences for the as-supplied specimen material.

For the rectangular beam shown in Fig. 1, the location of the centroid C in the undeformed position is $(x_c, \frac{h}{2}, \frac{t}{2})$ relative to the bottom left corner of the beam. The origin for the centroidal coordinate system can be chosen anywhere along the beam specimen. However, in practice, the origin is oftentimes chosen at one end of the beam specimen and oriented so that x_c is positive along the entire length of the beam.

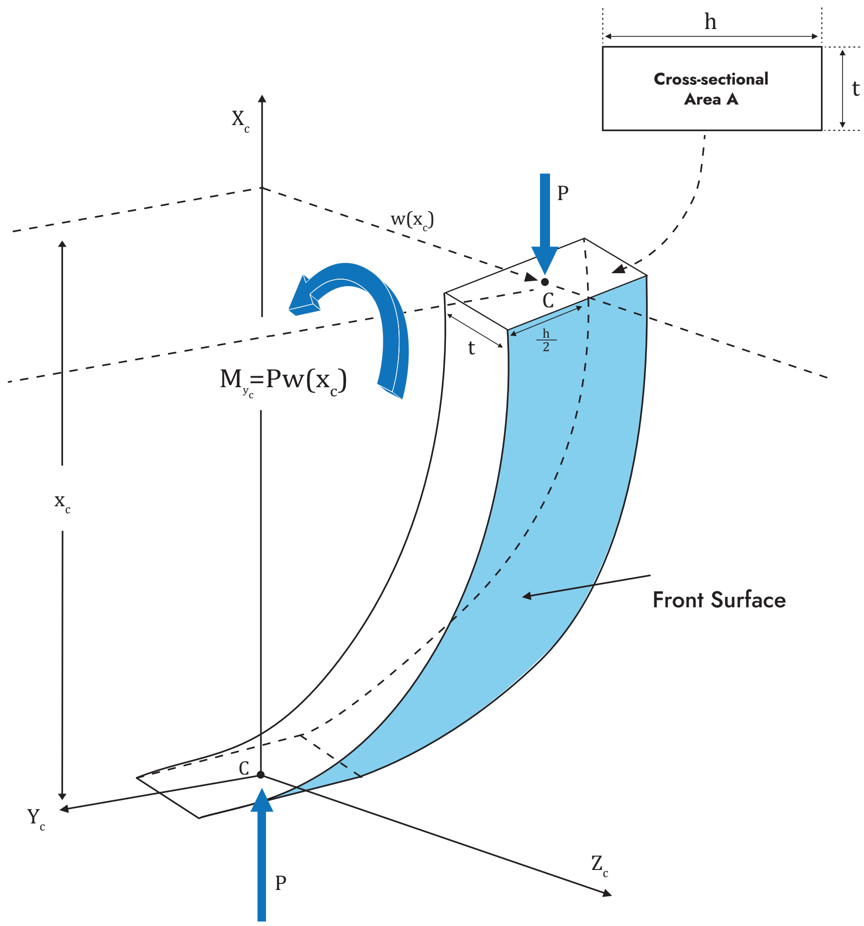

Figure 1: Beam-column undergoing compressive load with out-of-plane deflection in Z_c direction

Specimen dimensions: t = 0.03125 in, h = 1.00 in, L = 4 in, A = 0.03125 in^2

What is unique for a beam-column is that the bending moment M_{y_c} is solely due to the axial load P and the beam deflection, $w(x_c)$, as shown schematically in Fig. 1. If $w(x_c)$ is in the $+Z_c$ direction, then the moment about the Y_c axis is positive. Thus, we have that

(2) $$M_{y_c}(x_c) = P * w(x_c)$$

Combining Eqs. (1) and (2), the beam-column equation is given as;

(3) $$EI_{y_c y_c}\frac{d^2w}{dx_c^2}(x_c) = -P * w(x_c)$$

Eq. 3 is the well-known Euler buckling equation. The solution to Eq. 3 is

(4) $$w(x_c) = A\cos(\beta x_c) + B\sin(\beta x_c),\ \beta = \sqrt[2]{(P/EI_{y_c y_c})}$$

The amplitude parameters A and B are determined via the specified boundary conditions. In this laboratory, the boundary conditions are best approximated by assuming the following displacement conditions at the ends

(5) $$w(x_c = 0) = 0$$
$$w(x_c = L) = 0$$

Eqs. 4 and 5 show that $A = 0$ and that the lowest frequency, $\beta_{min} = \frac{\pi}{L}$. Furthermore, if the beam-column were perfectly straight, the compressive load that initiates buckling is given by $P_{crit} = \frac{\pi^2 EI_{YcYc}}{L^2}$

Of course, if the beam is even slightly bent, then there is no need for the buckling load as the beam already has deformed out-of-plane and will continue to bend out-of-plane with increasing load. If this is true, this is a simple example of a condition known as "post-buckling". In such a case, the final solution for the displacement is

(6) $$w(x_c) = B \sin\left(\frac{\pi x_c}{L}\right)$$

where B is an arbitrary amplitude for the beam that cannot be determined using the applied loading. If the beam deflection is measured at one location, say $w(x_c = \frac{L}{2})$, then the solution can be written;

(7) $$w(x_c) = w\left(\frac{L}{2}\right) * \sin\left(\frac{\pi x_c}{L}\right)$$

Thus, Eq. (7) shows that the theory predicts the beam shape is a sine function along the length, though the amplitude of the displacement cannot be determined unless it is measured at a point, such as $x_c = \frac{L}{2}$

In addition to Eq. (1), when cross-sections $Y_C - Z_C$ remain planar during mechanical loading, then static analysis of the beam and simple strength-of-materials concepts can be employed to show that there is a normal stress, $\sigma_{x_c x_c}$ on each cross-section given as;

(8) $$\sigma_{x_c x_c} = \frac{M_{y_c}(x_c) z_c}{I_{y_c y_c}} - \frac{P}{A}$$

where P is the compressive load and A is the cross-sectional area of the beam. If the beam is assumed to be loaded only on the top and bottom surfaces in the axial direction (see Figs. 1 and 3b) and the beam is an isotropic, homogeneous, linear elastic material, then the corresponding strains along the x_c direction can be written;

(9) $$\varepsilon_{x_c x_c} = \frac{M_{y_c}(x_c) z_c}{EI_{y_c y_c}} - \frac{P}{AE}$$

If the strain, $\varepsilon_{x_c x_c}$, is obtained experimentally on the front surface (see Fig. 1) at the same location as $w(x_C)$, say at $x_c = \frac{L}{2}$ then Eq. (9) can be re-written to solve for the axial load P as follows:

(10) $$\varepsilon_{x_c x_c}\left(\frac{L}{2}\right) = \frac{P w\left(\frac{L}{2}\right)\frac{t}{2}}{EI_{y_c y_c}} - \frac{P}{AE} = P\,\frac{w\left(\frac{L}{2}\right)\frac{t}{2}}{EI_{y_c y_c}} - \frac{1}{AE}$$

$$P = \frac{E\,\varepsilon_{x_c x_c}\left(\frac{L}{2}\right)}{\left[\frac{6w\left(\frac{L}{2}\right)}{ht^2} - \frac{1}{th}\right]}$$

Eq. 10 provides a theoretical formulation that can be used to predict the applied compressive load, P; by combining Eq. 10 with surface measurements, the axial compressive load can be predicted. The load obtained using Eq. 10 can be compared directly to the actual applied load as a way to assess the accuracy of the measured strain and out-of-plane displacement.

In contrast to the discussion given in Lab 2, for the post-buckling specimen the theory shows that we can predict the SHAPE of the $w(x_c)$ data but we cannot use the load to predict the *magnitude* of the displacements. Thus, we cannot *a priori* determine whether the experimental displacements are large enough to be measured by the VIC-EDU system ($|w(x_c)| > 0.04mm$).

NOTE **Throughout these labs, we refer to the VIC-EDU system in two ways. The hardware (case, cameras, cords, calibration targets) are referred to as "VIC-EDU" and the software (the special version of VIC-3D and VIC-Snap that are designed for this system) as "VIC-3D (EDU)" and "VIC-Snap (EDU)".**

III. EXPERIMENTAL PREPARATIONS

III. a. Specimen Preparation

Specimen preparations for the beam-column specimen are similar to those described in Lab 2, the VIC-EDU User's Manual [1] and the digital image correlation book [2]. To make accurate DIC measurements using images of a specimen, the specimen must be prepared and the FRONT SURFACE patterned with high contrast speckles having the appropriate size and an approximately random distribution across the field of view. The procedures to do this are detailed in LAB 1 and in Application Note AN-1701 [3]. The steps are briefly summarized.

- Use coarse and fine grit sandpaper to smooth surface and remove unwanted adherent materials.
- Use CSM-2 degreaser or similar cleaning agent to remove any residual oils and metal particles left on the surface by sanding. Repeat this as needed.
- Allow surface to dry and then paint the specimen surface. DO NOT paint specimen in same room where imaging is performed, as it will degrade the optical imaging elements. Typically, surface is initially painted white.
- Use only enough paint to coat surface and minimize reflections; DO NOT OVERPAINT SURFACE.
- After the white paint has dried, apply a random black dot pattern with size of 0.026" (0.635 mm) using the supplied roller and ink pad. Patterning should require approximately 2-3 passes in order to achieve the high contrast 50/50 ratio of white to black that is best. Do not try to align the roller passes, as attempts to move the roller around during patterning will smear or blur the dots, potentially degrading the quality of stereo DIC measurements. When done, image the front surface and check the pattern with VIC-3D (EDU) tools.

III. b. Installing Specimen in Loading System

Once the specimen has been prepared, the specimen can be placed in the load frame. One end of the specimen displaces downward due to the load. The other end of the specimen does not displace, both ends are free to rotate. These end conditions are SIMPLE SUPPORTS.

III. c. VIC-EDU System Preparation

Once the specimen is in the loading frame, the as-suppled tripod should be located in front of the specimen and the VIC-EDU system firmly attached to the tripod mounting head. At this point, the VIC-EDU system should be powered up and oriented to obtain images of the front surface of the specimen. Once the system is operational, the procedures outlined in Lab 1 for focusing the specimen and preparing to acquire images should be re-read and then used to locate the system approximately 0.5m in front of the specimen where focused images can be obtained. At this point, calibration of the system is performed.

III. d. VIC-EDU System Calibration

Once the VIC-EDU system is mounted to the tripod and placed in position where the specimen is in sharp focus, system calibration is performed. Though in principle the entire system could be picked up and moved carefully to a new location for calibration, and then moved back into position for viewing the specimen without affecting the measurements, such movements may introduce changes in the optical imaging system that are not readily apparent and can affect the accuracy of the results. To minimize the potential for such errors, it is recommended that you calibrate in front of the test specimen. There should be enough depth of field to allow for this. Lighting provided within the VIC-EDU measurement head should provide bright, clear images of the as-supplied calibration target.

When performing calibration, it is recommended that the target occupy 80-90% of the field of view. For this experiment, the as-supplied target dot pattern has black dots with a 14mm spacing on a white background. Fig. 4 shows a typical dot pattern imaged by the VIC-EDU system. Furthermore, it is required that all three of the black dots containing a central white dot be within the field of view of both cameras. These dots provide the necessary information for estimating rotation of the planar target.

To perform calibration, the target should be in focus for both cameras. Since the focus is fixed on the VIC-EDU system, the target should be located approximately 0.50m in front of the system. Then, the target is rotated several times and focused images acquired. The rotations should include motions about three separate orthogonal axes to ensure accurate identification of all calibration parameters. Typically, between 25 and 50 calibration images are acquired. Please refer to Lab 1 and the User's Manual for additional information regarding calibration and processing of the calibration images.

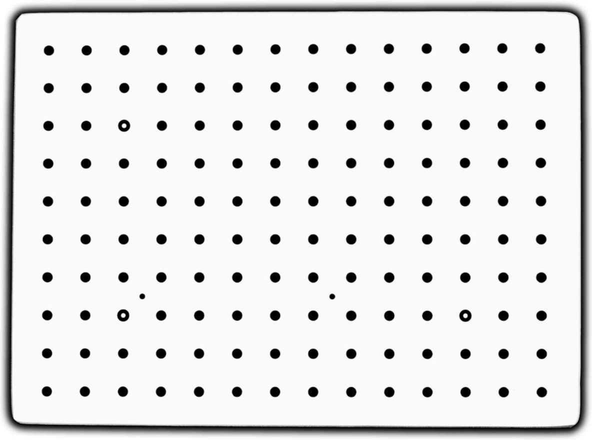

Figure 2: Schematic of a calibration target with 14mm dot spacing

IV. PERFORMING EXPERIMENT WITH IMAGE ACQUISITION

Once calibration is successfully completed in front of the specimen, the target is removed so the front surface of the beam specimen already placed in the loading frame is visible and in focus.

- Read Lab 1 and VIC-EDU User's Manual regarding the procedure for initiating VIC-EDU to acquire and store image pairs for this experiment.
- Acquire several image pairs of the speckled front surface of the specimen in the unloaded state.
- Apply the first increment of load/displacement and acquire two to three pairs of images, recording the image numbers and the corresponding applied load/displacement.
- Continue load/displacement application and image acquisition process until you reach the pre-identified maximum load or displacement for this specimen.
- Terminate the image acquisition process and the experiment via the process described in Lab 1 (e.g., close VIC-Snap (EDU).
- Unload the specimen.

At this point, the experimental process is complete and the deformation data is embedded in the image pairs that were acquired during the experiment. To extract full-field deformation data, the VIC-3D (EDU) software is initiated to analyze the images. Please refer to the detailed description in Lab 1 for how to initiate VIC-EDU and extract displacement and strain data.

V. POTENTIAL SOURCES OF DIFFERENCES BETWEEN PREDICTIONS AND MEASUREMENTS

For the beam-column specimen, the analytical results using Eq. 1 require you to assume specific boundary conditions at $x = 0$ and $x = L$. If the bottom support actually moves slightly outward/inward during loading at one end and has restricted rotation, then these will affect the comparison. If either of these occur, the entire $w(x_c)$ curve can be affected. Such motions are oftentimes difficult to remove from the experimental process and so may affect the comparison of experimental and theoretical displacements.

An additional source of potential errors is the way in which the experimental data is processed. For example, the experimental strains obtained from VIC-EDU use the measured displacements in a region, with a "finite-size strain filter" to estimate the strains at a point. The strain filter has various sizes over which it determines the strain. This process may introduce slight shifts in the strain relative to the actual strain at the corresponding point on the specimen. If $\varepsilon_{x_c x_c}$ is measured at $x = \frac{L}{2}$, then the size of the strain filter can affect the measured strain value and comparisons to theoretical estimates.

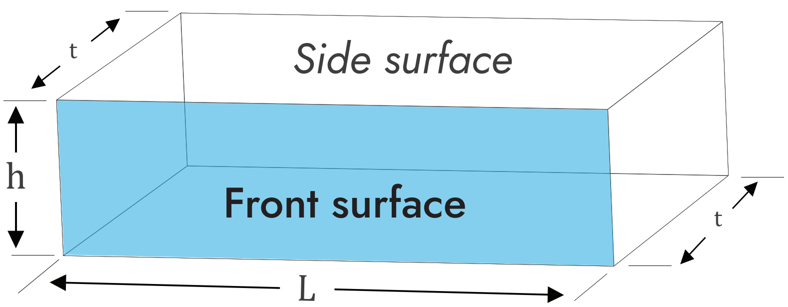

Figure 3a: General schematic of the beam-column specimen to be used with compressive loading to obtain out-of-plane displacements

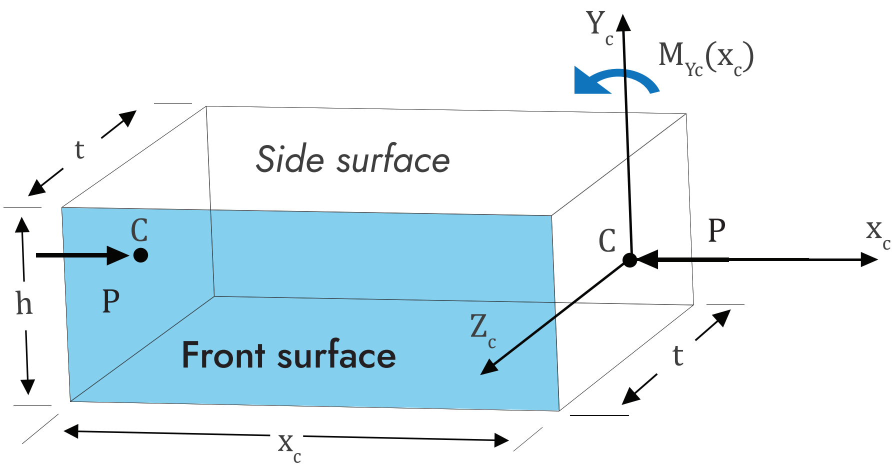

Figure 3b: Beam-column specimen with compressive load and corresponding bending moment. Front surface will deform in Z_c direction causing out-of-plane displacements

VI. LABORATORY RESULTS TO BE PRESENTED

Note: The range on the modulus of elasticity, E, for the test material is 3 GPa ≤ E ≤ 9 Gpa, with the expected value being 6 Gpa.

1. Measure the specimen geometry (L, h, t) several times with a micrometer or similar device and provide a table of the individual results, mean value for each dimension and standard deviation for each dimension.

2. Using the VIC-EDU system, acquire two pairs of test images of the as-patterned specimen after it has been centered, focused and properly exposed, but without any load. Both pairs of images should be taken without moving the specimen. Include one pair of test images for the beam-column specimen in the report.

 - As you are setting up the analysis, fill in the table below to provide a list of all options/parameters used in the analysis. Note that this table should be filled out for all StereoDIC experiments, without exception, to have a record of the analysis parameters being used.
 - Using one pair of stereo images obtained for $P = 0$, plot the initial shape of the specimen, $z(x, y)$, to see whether it is nominally flat or not. Here, it is particularly important to show the initial shape and determine whether the specimen is in a "post-buckled" shape that is bent towards or away from the cameras.
 - Discuss how the measurements are related to the concept of "post-buckled shape".

3. Use two pair of stereo images obtained for P=0 and measure out-of-plane displacement field, $w(x_c, y_c \frac{t}{2})$, of the specimen on the front surface in Fig. 1. For the remainder of this section z= $\frac{t}{2}$ is assumed to be true, so $w(x_c, y_c)$ $= w(x_c, y_c \frac{t}{2})$.

 - Plot $w(x_c, y_c = 0)$ along the length of beam, which is along the x_c-axis in Fig. 1
 - Obtain the mean value for $w(x_c, 0)$ and the standard deviation in $w(x_c, 0)$ along centerline
 - Discuss the results and relationship to CSI estimate of 0.04mm for variability.

4. Assuming the experimental beam-column boundary conditions are consistent with Eq. (5), the beam-column displacement data is theoretically given by Eq. (7).

 - Use VIC-3D (EDU) analysis software to extract the out-of-plane displacement data along the x_c-axis direction in Fig. 1, which is along the beam length that passes through the centroid location for $0 \leq x_c \leq L$ for N compressive loads.
 - Extract $w(x_c = \frac{L}{2}, y_c = 0)$ from the measurements for each load and insert into Eq. (7) for each load.
 - Plot Eq. (7) vs the experimental data on $0 \leq x_c \leq L$ for compressive loads P_i, i = 1, 2, ..., N.

5. Using the VIC-3D (EDU) analysis software, extract the strain $\varepsilon_{x_c x_c}$ along the x_c-axis direction on the front surface in Fig. 1, which is along the beam length, for $0 \leq x_c \leq L$ for N compressive loads (same line as used in items 2-4).
 - Extract $\varepsilon_{x_c x_c}(x_c = \frac{L}{2}, 0)$ from the front surface strain measurements for each load.
 - Using the measured values of $w(x_c = \frac{L}{2}, 0), \varepsilon_{x_c x_c}(\frac{L}{2}, 0), t, L, h$ for each load, calculate the compressive load using Eq. (10).
 - Plot the ratio $[\frac{P_{Eq.(10)}}{P_{true}}]\ vs\ P_{true}$

Undergraduate Students Only

6. Discuss results from 1., 2., and 3.

Lab 3 - Stereo DIC Image Analysis Parameters

Region of Interest (ROI) (pixels x pixels)	
Field of View (pixels x pixels)	
Digital magnification (pixels/mm)	
Subset size (pixels)	
Subset spacing (pixels)	
Correlation criteria used	
Subset weighting	
Interpolation method	
Consistency threshold (pixels)	
Confidence margin (pixels)	
Matching threshold (pixels)	
Strain metric type	
Strain filter size (N x N pixels^2)	
Strain filter type	
Auto-plane fit	Yes or No

Graduate Students

7. This part compares the strains computed using VIC-3D (EDU) to the values obtained using a simple formula from undergraduate solid mechanics. You already have determined strain with VIC-3D (EDU) along the x-axis in Fig. 1 through the centroid.
   - Along the x_c-axis direction in Fig. 1, output the axial displacement, $U(x_c,0) = U(x_c)$, obtained for each subset at the center of the front surface.
   - For each neighboring value, compute $E_{x_c x_c} = \dfrac{U(x_c + \Delta x_c) - U(x_c)}{\Delta x_c}$ for all the points along the line for each load.
   - Plot $\varepsilon_{x_c x_c}$ and $E_{x_c x_c}$ along $0 \le x_c \le L$ for N compressive loads.

8. The results obtained by these methods will not agree well. However, if you calculate strain $e_{x_c x_c}$ using the following approach, then it should agree with $\varepsilon_{x_c x_c}$.
   - Along the line, define the undeformed coordinates of a point as (X_c, Y_c, Z_c) in the unloaded state for each point of interest.
   - For each pair of neighboring subsets, compute the undeformed distance between the 2 subsets as $\delta = \sqrt[2]{(X_{c_{i+1}} - X_{c_i})^2 + (Y_{c_{i+1}} - Y_{c_i})^2 + (Z_{c_{i+1}} - Z_{c_i})^2}$
   - For each pair of neighboring subsets and load P_i, let $(x_{c_i}, y_{c_i}, z_{c_i})$ denote the deformed position of the originally undeformed point $(X_{c_i}, Y_{c_i}, Z_{c_i})$. Compute the deformed length of the distance between the same subsets as $\Delta = \sqrt[2]{(x_{c_{i+1}} - x_{c_i})^2 + (y_{c_{i+1}} - y_{c_i})^2 + (z_{c_{i+1}} - z_{c_i})^2}$ for all the points along the line for each load.
   - Compute $e_{x_c x_c} = \dfrac{\Delta - \delta}{\delta}$ for all subset pairs.
   - Plot $\varepsilon_{x_c x_c}$ from the software and $e_{x_c x_c}$ along $0 \le x_c \le L$ for N compressive loads.

If done correctly, these results will agree well and this should be evident in the comparison plot. Please discuss these results and explain the difference between $\varepsilon_{x_c x_c}$ and $E_{x_c x_c}$.

9. If the maximum applied load is P_{max}, show the full-field plot of (x_c, y_c) on the entire front surface for $0 \le x_c \le L$ for $0, \frac{P_{max}}{5}, \frac{2P_{max}}{5}, \frac{3P_{max}}{5}, \frac{4P_{max}}{5}$ and P_{max}.
   - Is the strain field constant along the length or does it vary as you move from 0 to L?
     - If it varies, how does it vary?
     - Please explain why the observations make sense using the theoretical results from Eqs. (1-10).
       - Does the strain field vary across the width of the specimen?
     - Please explain your findings based on Eqs. (1-10).

10. Discuss results in 1., 2., 3., 4. and 5.

NOTES

VII. REFERENCES

1. VIC-EDU User's Manual, Correlated Solutions Incorporated, www.correlatedsolutions.com
2. MA Sutton, JJ Orteu, and HW Schreier, Image Correlation for Shape Motion and Deformation Measurements; Theory and Applications, Springer (2009) ISBN: 978-0-387-78746-6.
3. Application Note AN-1701:Speckle Pattern Fundamentals, http://www.correlatedsolutions.com/support/index.php?/Knowledgebase/Article/View/80/1/speckle-pattern-fundamentals.

Laboratory 4

Disk in Diametral Compression Using StereoDIC Imaging Systems and VIC-EDU

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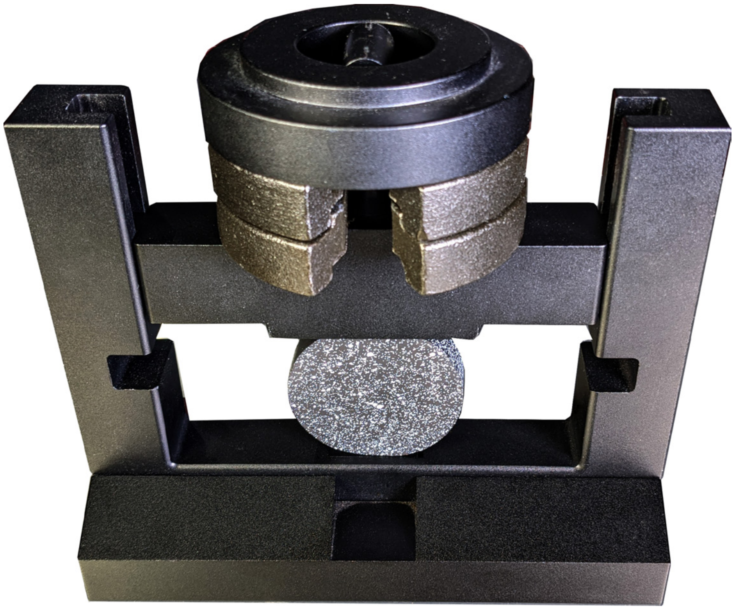
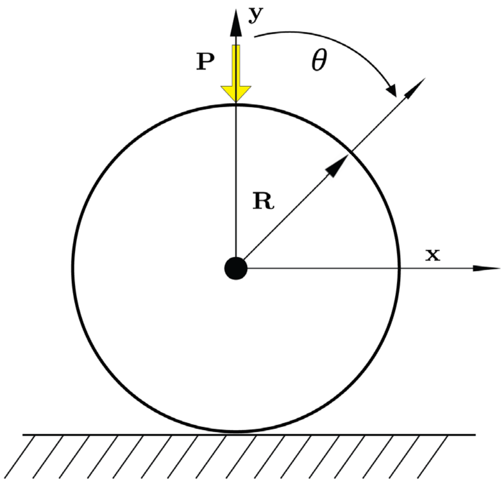


I. OVERVIEW

The purpose of this laboratory is to utilize optical imaging systems with stereo digital image correlation within VIC-EDU [1] to make full-field, non-contacting surface deformation measurements with the educational measurement system for a circular disk loaded in diametral compression. As part of this laboratory, students will be provided basic theoretical background to predict the surface displacements and strains that they will measure on the disk. To obtain the measurements, students will learn how to (a) prepare the specimen surface and apply a usable, high-contrast speckle pattern, (b) arrange the VIC-EDU system to acquire images that can be used for stereo analysis, (c) calibrate the VIC-EDU system for stereo DIC, (d) acquire and store images during mechanical loading of the specimen, (e) perform post-processing to obtain results using the stereo images and (f) compare the experimental results to the theoretical solutions.



central $x - y$ coordinate system

II. DISK IN DIAMETRAL COMPRESSION THEORY

A schematic diagram for a disk in diametral compression is shown in Figure 1. For an isotropic, homogeneous, elastic material, there are existing theoretical solutions for the deformations of the disk [2]. Figure 2 shows the variables used in the solutions obtained in [2]. Of particular import is the angle θ. As shown in Figs.1 and 2, the angle is defined as positive clockwise from the y-axis. This definition is different from most applications, so it is important to be aware of this definition. Because of this definition, the position of an arbitrary point, p, is (r, θ) in a cylindrical polar coordinate system and $(x = r\, sin(\theta), y = r\, cos(\theta))$ in a centrally located $x - y$ system.

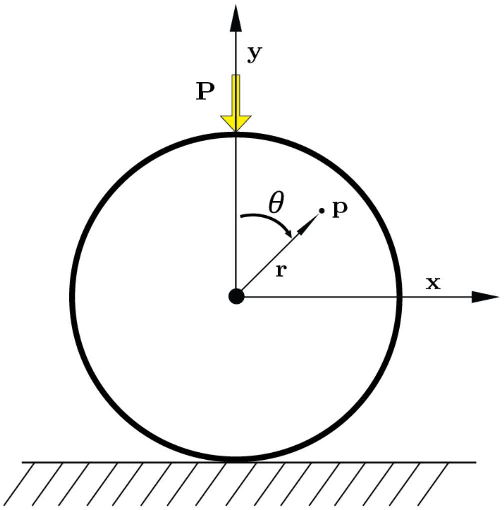
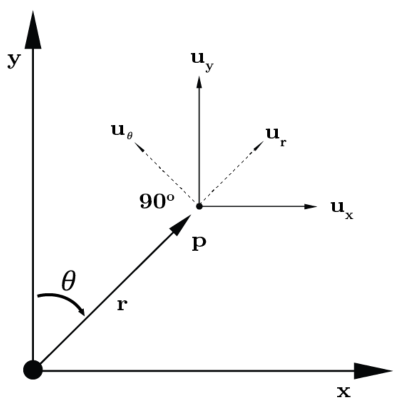


Figure 2a: Disk in diametral compression with angle θ and radial distance r for arbitrary point, p, in the disk



$$\theta = \tan^{-1}(x/y)$$

Figure 2b. Definition of angle, θ; normalized radial position, ρ=r/R; displacement components (u_r, u_θ) in the $r - \theta$ coordinate system; displacement components (u_x, u_y) in the $x - y$ coordinate system

The general expressions for the radial and angular displacements were obtained in previous work [1] and are given in the Appendix. Note that the expressions in the Appendix are for the special case where both sides of the disk move in the vertical direction. The predictions for the vertical displacement have been slightly modified in the remainder of this laboratory to account for a fixed point at the bottom of the specimen.

For a horizontal line along the + x-axis with y = 0, angle $\theta = +\frac{\pi}{2}$ and $0 \le \rho \le 1$, the displacement components (u_x, u_y) can be written as follows;

(1)
$$u_x\left(\rho,\frac{\pi}{2}\right) = -\frac{2P}{E\pi t}\left\{-\frac{\rho}{\rho^2+1}[(\rho^2+3)-v(1-\rho^2)]+2(1-v)\tan^{-1}\rho\right\}$$
$$u_y(\rho,\tfrac{\pi}{2}) = C(P)$$

$C(P)$= vertical displacement of center-point located at (0,0) for load P.

For the horizontal line through center-point along the -x-axis with y = 0, angle $\theta = -\frac{\pi}{2}$ and $0 \le \rho \le 1$, the displacement components (u_x, u_y) can be written as follows;

(2)
$$u_x\left(\rho,-\frac{\pi}{2}\right) = -\frac{2P}{E\pi t}\left\{-\frac{\rho}{\rho^2+1}[(\rho^2+3)-v(1-\rho^2)]+2(1-v)\tan^{-1}\rho\right\}$$
$$u_y(p,\tfrac{-\pi}{2}) = C(P)$$

where P is the applied load, $\rho = \frac{r}{R}$,R is the outer radius of the disk, r is the radius to an arbitrary point in the disk, θ is the angle defined in Figs 1 and 2, t is the thickness of the disk, E is the elastic modulus for the disk, ν is Poisson's ratio and G is the shear modulus for the disk. The specimen provided with the VIC-EDU system has an estimated modulus of elasticity, E ~ 20 GPa and ν ~ 0.40. The shear modulus, $G = E/2(1+\nu)$, is determined from the estimated values for E and ν. As described in a NOTE in Section VI of this Lab, there may be variability in E due to manufacturing differences for the as-supplied specimen material. Here, $\tan^{-1} 0 = 0$ for $\rho = 0$ along x-axis and $\tan^{-1}(1) = \frac{\pi}{4}$ at the right edge of the disk where $\rho = \frac{R}{R} = 1$. The parameter, $C(P)$, can be viewed as the total downward displacement at point (0,0) of the bottom half of the cylindrical disk ($\frac{\pi}{2} \le \theta \le \frac{3\pi}{2}$) due to the applied load, P.

By symmetry of the loaded specimen in the (r, θ) system about the y-axis , the strains along the +x-axis ($\theta = \frac{\pi}{2}$) and -x-axis ($\theta = -\frac{\pi}{2}$) can be written as follows:

(3)
$$\varepsilon_{xx}\left(\rho,\frac{\mp\pi}{2}\right) = \frac{P}{E\pi Rt}\left\{\left(\frac{1-\rho^2}{1+\rho^2}\right)^2 - v\left[\frac{\rho^8+4\rho^6+2\rho^4-4\rho^2-3}{(1+\rho^2)^4}\right]\right\}$$
$$\varepsilon_{yy}\left(\rho,\frac{\mp\pi}{2}\right) = \frac{P}{E\pi Rt}\left\{\left[\frac{\rho^8+4\rho^6+2\rho^4-4\rho^2-3}{(1+\rho^2)^4}\right] - v\left(\frac{1-\rho^2}{1+\rho^2}\right)^2\right\}$$

For a vertical line along the $+y$-axis with x = 0, the elastic displacements are as follows:

(4)
$$u_r(\rho,0) = u_y(\rho,0) = -\frac{2P}{E\pi t}\left\{-(1-v)\rho + 2\ln\left|\frac{1+\rho}{1-\rho}\right|\right\} - |C(P)|$$
$$u_\theta(\rho,0) = u_x(\rho,0) = 0$$

For a vertical line along the $+y$-axes with $x = 0$, $\varepsilon_{xy} = \varepsilon_{r\theta} = 0$ and the elastic strains are;

$$\varepsilon_{rr}(\rho,0) = \varepsilon_y(\rho,0) = -\frac{P}{E\pi Rt}\left\{\frac{3+\rho^2}{1-\rho^2}+v\right\}$$

(5) $$\varepsilon_{\theta\theta}(\rho,0) = \varepsilon_x(\rho,0) = \frac{P}{E\pi Rt}\left\{1+v\frac{3+\rho^2}{1-\rho^2}\right\}$$

$$\varepsilon_{r\theta}(\rho,0) = \varepsilon_{xy}(\rho,0) = 0$$

Similar results are also obtained for all strains for the $-y$ axis (i.e., where $\theta = \pi$).

III. EXPERIMENTAL PREPARATIONS

NOTE Throughout these labs, we refer to the VIC-EDU system in two ways. The hardware (case, cameras, cords, calibration targets) are referred to as "VIC-EDU" and the software (the special version of VIC-3D and VIC-Snap that are designed for this system) as "VIC-3D (EDU)" and "VIC-Snap (EDU)".

III.a. Specimen Preparation

To make accurate DIC measurements using images of a specimen, the specimen must be prepared and the FRONT SURFACE patterned with high contrast speckles having the appropriate size and an approximately random distribution across the field of view. The procedures to do this are detailed in Lab 1 and in Application Note AN-1701 [3] and are briefly summarized below:

- Use coarse and fine grit sandpaper to smooth surface and remove unwanted adherent materials
- Use CSM-2 degreaser or similar cleaning agent to remove any residual oils and metal particles left on the surface by sanding. Repeat this as needed
- Allow surface to dry and then paint the specimen surface. DO NOT paint specimen in same room where imaging is performed, as it will degrade the optical imaging elements. Typically, surface is initially painted white
- Use only enough paint to coat surface and minimize reflections; DO NOT OVERPAINT SURFACE
- After the white paint has dried, apply a random black dot pattern with size of 0.026" (0.635 mm) using the supplied roller and ink pad. Patterning should require approximately 2-3 passes in order to achieve the high contrast 50/50 ratio of white to black that is best. Do not try to align the roller passes, as attempts to move the roller around during patterning will smear or blur the dots, potentially degrading the quality of stereo DIC measurements. When done, image the front surface and check the pattern with VIC-3D (EDU) tools.

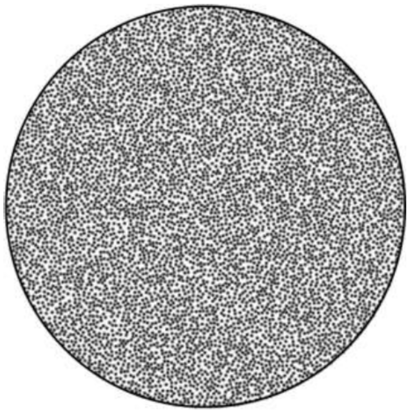


Figure 3: Image of a typical patterned disk specimen

III.b. Installing Specimen in Loading System

Once the specimen has been prepared and placed in the mechanical loading frame, the loading and displacement conditions used in the theory should be approximated as best possible. Thus, when setting up the experiment, care should be taken to try and configure the loading and support conditions to approximate these conditions. These BCs are as follows;

- Vertical compressive loading will be applied on top surface along the y-axis, so $F_y = -P$
  - Loading should be applied equally across the entire disk thickness so that the front surface is approximately in a plane stress condition
- Bottom contact point on disk does not have any displacement so that the displacement (d_x, d_y) at the bottom of the disk is $(0,0)$.

III.c. VIC-EDU System Preparation

Once the specimen is in the loading frame, the as-suppled tripod should be located in front of the specimen and the VIC-EDU system firmly attached to the tripod mounting head. At this point, the VIC-EDU system should be powered up and oriented to obtain images of the front surface of the specimen. Once the system is operational, the procedures outlined in Lab 1 and the User's Manual for focusing the specimen and preparing to acquire images should be re-read and then used to locate the system in a position where focused images can be obtained. At this point, calibration of the system should be performed.

III.d. VIC-EDU System Calibration

Once the VIC-EDU system is mounted to the tripod and placed in position where the specimen is in sharp focus, system calibration is performed. Though in principle the entire system could be picked up and moved carefully to a new location for calibration, and then moved back into position for viewing the specimen without affecting the measurements, such movements may introduce changes in the optical imaging system that are not readily apparent and can affect the accuracy of the results. To minimize the potential for such errors, it is recommended that you calibrate in front of the test specimen. There should be enough depth of field to allow for this. Lighting provided within the VIC-EDU measurement head should provide bright, clear images of the as-supplied calibration target.

When performing calibration, it is recommended that the target occupy 80-90% of the field of view. For this experiment, the as-supplied target dot pattern has black dots with a 14mm spacing on a white background. Fig. 4 shows a typical dot pattern imaged by the VIC-EDU system. Furthermore, it is required that all three of the black dots containing a central white dot be within the field of view of both cameras. These dots provide the necessary information for estimating rotation of the planar target.

To perform calibration, the target should be in focus for both cameras. Since the focus is fixed on the VIC-EDU system, the target should be located approximately 0.50m in front of the system. Then, the target is rotated several times and focused images acquired. The rotations should include motions about three separate orthogonal axes to ensure accurate identification of all calibration parameters. Typically, between 25 and 50 calibration images are acquired. Please refer to Lab 1 and the User's Manual for additional information regarding calibration and processing of the calibration images.

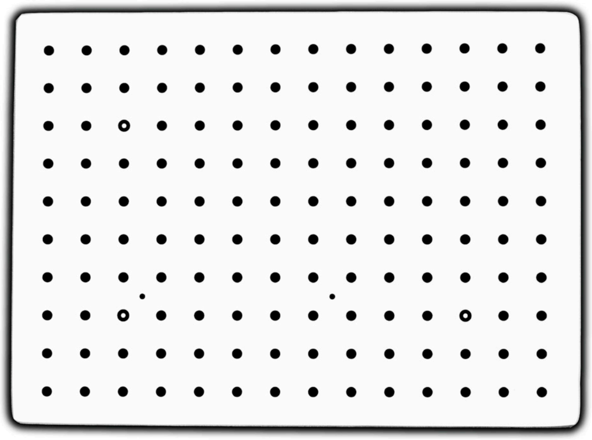


Figure 4: Photograph of a typical calibration target used to calibrate stereo-vision measurement systems such as VIC-EDU

IV. PERFORMING EXPERIMENT WITH IMAGE ACQUISITION

Once calibration is successfully completed in front of the specimen, the target is removed so the front surface of the beam specimen already placed in the loading frame is visible and in focus.

- Read Lab 1 and VIC-EDU User's Manual regarding the procedure for initiating VIC-EDU to acquire and store image pairs for this experiment.
- Acquire several image pairs of the speckled front surface of the specimen in the unloaded state.
- Apply the first increment of load/displacement and acquire two to three pairs of images, recording the image numbers and the corresponding applied load/displacement.
- Continue load/displacement application and image acquisition process until you reach the pre-identified maximum load or displacement for this specimen.
- Terminate the image acquisition process and the experiment via the process described in Lab 1 (e.g., close VIC-Snap (EDU).
- Unload the specimen.

At this point, the experimental process is complete and the deformation data is embedded in the image pairs that were acquired during the experiment. To extract the full-field deformation data, the VIC-3D (EDU) software is initiated to analyze the images. Please refer to the detailed description in Lab 1 for how to initiate VIC-3D (EDU) and extract displacement and strain data using the software functions available in the code.

V. IMPLICATIONS OF THEORY FOR DISK FRONT SURFACE MEASUREMENTS

With symmetry in the (r, θ) system about the y-axis, all points along the y-axis are predicted to have $u_x(0, y) = 0$ as shown in Eq (4). This can be checked experimentally by simply plotting u_x along the y-axis from the top to the bottom of the disk; any deviation of the u_x measurements from 0 should be randomly varying around 0. However, if there is a clear trend or a translation of the disk in the x-direction, this would indicate that motions other than those predicted by the theory have occurred. In fact, such motions often arise in actual experiments so that the specimen does not just deform due to load, but also moves and rotates due to variations in the boundary conditions. Such motions are known as **Rigid Body Motions (RBMs)** and are quite common in experiments. The presence of RBMs may need to be accounted for when looking to compare the experimental and theoretical displacements. This may be the cause for any motions at the bottom of the disk since the supports under the disk may deform or even rotate slightly, adding rigid body motions to points on the disk.

NOTE

You can remove rigid body motion from the displacement measurements using a "rigid body motion removal" function in the software. Please use this function as needed to see if there has been rigid body motion. Once the rigid body motion has been removed, then the resulting data should be due to true deformation of the specimen and hence consistent with the theoretical results.

Inspection of Eqs. 4 and 5 shows that, for point loads at the top and bottom of the specimen, the displacement and strain at a point under the load are singular. The theoretical predictions near the point load and the bottom contact point will not be accurate in our experiments since the load will be distributed over a small area on the top (and bottom) surfaces when load is applied, making the results non-singular at these points.

As noted earlier, physical boundary conditions require that the bottom point on the disk does not displace. To account for this boundary condition, the parameter, $C(P)$, in Eq. 4 is required. For the displacement component, u_y, $C(P)$ can be interpreted as the vertical displacement of all points along the x-axis shown in Figs. 1 and 2. Thus, this is the displacement of the top of the bottom half of the disk, designated as $-\delta$. Since the load point on the disk also displaces downward relative to the x-axis by the same amount as the bottom half of the disk deflects downward, then the displacement of the load point on the disk will be the sum of both displacements, $u_y(1,0) = -2\delta$.

NOTE

You can check the "assumed rigid" lower boundary by outputting displacement data for a subset located just above the contact point at the bottom of the disk. You can also check on the displacement just below the loading point. Displacements at a point just BELOW the loading point can be measured and then compared to the theoretical prediction at the same point for the same load. Here, the displacement should be $2\,C(P)$.

Another potential source of error is that finite-sized subsets are used to obtain the displacement at the center-point of the subset. This process will average the displacements over the subset size and thus may introduce slight shifts in the VIC-EDU measurement, especially in regions where there are high gradients in the displacements. Typically, the averaging process tends to underestimate the subset center-point displacement.

Interestingly, RBM does not affect the comparison of experimental and predicted strains, since strains in VIC-EDU are defined in a way that eliminates the effect of RBM on its value. So, even if there is RBM that affects the comparison of displacements, the measured and predicted strains such as ε_{xx} should be comparable. Issues that might affect the quality of the comparison include slight shifts in the load position relative to the assumed location $(0, R)$, accuracy of the measured geometric lengths and accuracy of the assumed modulus of elasticity, as well as other factors that are not noted here (e.g. temperature changes during the experiment).

An additional source of potential error is the way in which the experimental data is processed. For example, the experimental strains obtained from VIC-EDU use the measured displacements in a region, with a "finite-size strain filter" to estimate the strains at a point. The strain filter has various sizes over which it acquires the strain. This process may introduce slight shifts in the strain relative to the actual strain at the corresponding point on the specimen.

VI. LABORATORY RESULTS TO BE PRESENTED

1. Measure the specimen geometry (d, t) several times with a micrometer or similar device and provide a table of the individual results, mean value for each dimension and standard deviation for each dimension.

The range on the modulus of elasticity, E, for the disc material is 10 MPa ≤ E ≤ 30 Mpa, with a best estimate of 20Mpa. For ν, the indicated range is 0.33 ≤ ν ≤ 0.47, with a best estimate of 0.40.

2. Use two sets of stereo images obtained with P = 0 and obtain full-field strain data for ε_{yy} using the subset size and subset spacing to be used in future analyses and show plot.

- Obtain mean value and standard deviation in experimentally measured strain for the points where strain was obtained in the disk.
- Use these values to discuss how the observed variability will impact results.

3. Estimate the location of the disk center-point and define both y and x axes in the images having this center-point as the origin. For six applied loads;

$$P = 0, \frac{P_{max}}{5}, \frac{2P_{max}}{5}, \frac{3P_{max}}{5}, \frac{4P_{max}}{5} \text{ and } P_{max}$$

(a) Output full-field plots of experimentally determined u_x and u_y.

- Looking at all the plots, determine whether there is SYMMETRY in the data relative to the y-axis and discuss what you observe in the graphs.
- Looking at a point near the bottom of the disk (x = 0, $y \approx -R$), determine whether the displacements are consistent with the assumed boundary condition or not.

Remember, you can use the rigid body removal option in VIC-EDU and see how the displacement results compare before and after removing rigid body rotations and displacements

(b) Output the strain ε_{yy} along the line (0, y) for all y in the range $-\frac{R}{2} \leq y \leq +\frac{R}{2}$

- Compare the strain along this line to the theoretical prediction for each load using mean values in the geometric parameters.

(c) Output the strain ε_{xy} along the line (0, y) for all y in the range $-\frac{R}{2} \leq y \leq +\frac{R}{2}$

- Compare the strain along this line to the theoretical prediction for each load. Note that the theory predicts no shear strain along a line of symmetry using mean values in the geometric parameters.

You can use the rigid body removal option in VIC-EDU again, and see how the strains results compare before and after removing rigid body motions. Note that, theoretically, the strain fields should be the same either way.

4. At a point near the top of the disk (e.g., near (0, R)), output the vertical displacement for ALL loads. Similarly, for a point at the center of the disk (e.g., subset with center at approximately (x = 0, y = 0)), output the vertical displacement for ALL loads.

- Plot P vs measured $u_y(0, R)$ and $u_y(0,0)$ on the same graph and compare these results by looking at ratio $\frac{u_y(0, R)}{u_y(0, 0)}$ for each load, P. Discuss what you observed.

If the previous displacement results required rigid body removal to agree with the theoretical predictions, you will need it here. So, it would be interesting to plot the results before and after removing rigid body rotations and displacements.

5. For six applied loads, $P = 0, \frac{P_{max}}{5}, \frac{2P_{max}}{5}, \frac{3P_{max}}{5}, \frac{4P_{max}}{5}$ $and\ P_{max}$, output the experimental strain $\varepsilon_{xx}\,(x, \mathrm{y} = 0)$ for $-R \leq x \leq +R$

- Calculate the therorετical $\varepsilon_{xx}\,(x, 0)$ for $-R \leq x \leq +R$ and compare the theory and experimental strains for the six loads.

Discuss the results shown above, with special emphasis on whether there is evidence of rigid body motion or not in any of the comparisons.

VII. APPENDIX

General Expressions for Elastic Displacements and Strains for Disk in Diametral Compression

The displacements u_r and u_θ at any point in a diametrically loaded disk subjected to point load P were derived in [2] and are written as follows;

$$u_r = -\frac{2P}{\pi}\left\{\begin{aligned}&\left[\frac{\rho[2+\rho^2+\rho^4-(1+3\rho^2)\cos 2\theta]}{1+\rho^4-2\rho^2\cos 2\theta} + \sin\theta\left(\tan^{-1}\frac{\rho\sin\theta}{1+\mathrm{p}\cos\theta}+\tan^{-1}\frac{\rho\sin\theta}{1-\rho\cos\theta}\right)+\cos\theta\ln\left|\frac{1+\rho^2+2\rho\cos\theta}{1+\rho^2-2\rho\cos\theta}\right|\right]\\ &+\mathrm{v}\left[\frac{\rho(1-\rho^2)(\rho^2-\cos 2\theta)}{1+\rho^4-2\rho^2\cos 2\theta}\sin\theta\left(\tan^{-1}\frac{\rho\sin\theta}{1+\rho\cos\theta}+\tan^{-1}\frac{\rho\sin\theta}{1-\rho\cos\theta}\right)\right]\end{aligned}\right\}$$

$$u_\theta = -\frac{2P}{\pi}\left\{\begin{aligned}&\left[\cos\theta\left(\tan^{-1}\frac{\rho\sin\theta}{1+\mathrm{p}\cos\theta}+\tan^{-1}\frac{\rho\sin\theta}{1-\rho\cos\theta}\right) - \sin\theta\left(\frac{2\rho(1-\rho^2)\cos\theta}{1+\rho^4-2\rho^2\cos 2\theta}+\ln\left|\frac{1+\rho^2+2\rho\cos\theta}{1+\rho^2-2\rho\cos\theta}\right|\right)\right]\\ &-\mathrm{v}\cos\theta\left[\frac{2\rho(1-\rho^2)\sin\theta}{1+\rho^4-2\rho^2\cos 2\theta} + \tan^{-1}\frac{\rho\sin\theta}{1+\rho\cos\theta}+\tan^{-1}\frac{\rho\sin\theta}{1-\rho\cos\theta}\right]\end{aligned}\right\} \qquad \text{(A-1)}$$

where P is the applied load, R is the outer radius of the disk, r is the radius to an arbitrary point in the disk, $\rho = \frac{r}{R}$, θ is the angle defined in Fig. 2, t is the thickness of the disk, E is the elastic modulus for the disk, ν is Poisson's ratio and G is the shear modulus for the disk. If the displacements are to be obtained in the $x - y$ system, then the same displacement in the $x - y$ system is written;

(A-2)
$$\begin{aligned} u_x &= u_r \sin\theta - u_\theta \cos\theta \\ u_y &= u_r \cos\theta + u_\theta \sin\theta \end{aligned}$$

Eqs. A-1 and A-2 must be used to obtain the displacements (u_x, u_y) in the disk for comparison to the theoretical predictions, since VIC-EDU reports displacements in the $x - y$ system.

Regarding the strains in the disk, it also was shown in [2] that the strains in the $r - \theta$ system are as follows.

(A-3)
$$\varepsilon_{rr} = \frac{P}{E\pi Rt}\left\{\frac{(1-\rho^2)^2(\rho^4+2\rho^2-1-2\cos 2\theta)}{(\rho^4+1-2\rho^2\cos 2\theta)^2}\right\} - \nu\frac{P}{E\pi Rt}\left\{\frac{\rho^8+4\rho^4-4\rho^2-1+2(-2\rho^6+\rho^4+1)\cos 2\theta}{(\rho^4+1-2\rho^2\cos 2\theta)^2}\right\}$$

$$\varepsilon_{\theta\theta} = \frac{P}{E\pi Rt}\left\{\frac{\rho^8+4\rho^4-4\rho^2-1+2(-2\rho^6+\rho^4+1)\cos 2\theta}{(\rho^4+1-2\rho^2\cos 2\theta)^2}\right\} - \nu\frac{P}{E\pi Rt}\left\{\frac{(1-\rho^2)^2(\rho^4+2\rho^2-1-2\cos 2\theta)}{(\rho^4+1-2\rho^2\cos 2\theta)^2}\right\}$$

$$\varepsilon_{r\theta} = \frac{P}{G\pi Rt}\left\{\frac{2(1-\rho^4)(1-\rho^2)\sin 2\theta}{(\rho^4+1-2\rho^2\cos 2\theta)^2}\right\}$$

where E, is modulus of elasticity, ν is Poisson's ratio and G is the shear modulus with

$$G = \frac{E}{2(1+\nu)}$$

The transformation of strains from the $r - \theta$ system to the $x - y$ system follows the general process outlined in undergraduate mechanics of materials. However, here there is a slight change in the equations since the angle θ is different than is commonly used. The strains in the $x - y$ system are as follows.

(A-4)
$$\begin{aligned} \varepsilon_{xx} &= \varepsilon_{rr}\sin^2\theta + \varepsilon_{\theta\theta}\cos^2\theta - 2\varepsilon_{r\theta}\sin\theta\cos\theta \\ \varepsilon_{yy} &= \varepsilon_{rr}\cos^2\theta + \sin^2\theta\,\varepsilon_{\theta\theta} + 2\varepsilon_{r\theta}\sin\theta\cos\theta \\ \varepsilon_{xy} &= -1 * [(\varepsilon_{rr} - \varepsilon_{\theta\theta})\sin\theta\cos\theta + \varepsilon_{r\theta}(\sin^2\theta - \cos^2\theta)] \end{aligned}$$

It is important to emphasize that Eqs. (A-3) and (A-4) must be used to convert the $r - \theta$ strains into the $x - y$ strains as $x - y$ strains are reported by VIC-3D (EDU) when analyzing the experimental images of the loaded calibration disk. Furthermore, if the strains are obtained using Eqs. (A-3) and (A-4), then the definition of the angle θ requires that the shear strain equation be multiplied by –1 to obtain the same sign as is used in VIC-EDU. The –1 factor already is included in the shear strain formula in Eq. (A-4).

Lab 4 - Stereo DIC Image Analysis Parameters

Region of Interest (ROI) (pixels x pixels)	
Field of View (pixels x pixels)	
Digital magnification (pixels/mm)	
Subset size (pixels)	
Subset spacing (pixels)	
Correlation criteria used	
Subset weighting	
Interpolation method	
Consistency threshold (pixels)	
Confidence margin (pixels)	
Matching threshold (pixels)	
Strain metric type	
Strain filter size (N x N pixels^2)	
Strain filter type	
Auto-plane fit	Yes or No

NOTES

VIII. REFERENCES

1. VIC-EDU User's Manual, Correlated Solutions Incorporated, www.correlatedsolutions.com
2. MA Sutton, JJ Orteu, and HW Schreier, Image Correlation for Shape Motion and Deformation Measurements; Theory and Applications, Springer (2009) ISBN: 978-0-387-78746-6.
3. Application Note AN-1701:Speckle Pattern Fundamentals, http://www.correlatedsolutions.com/support/index.php?/Knowledgebase/Article/View/80/1/speckle-pattern-fundamentals.